# The Macro Impact of the Debt-Inflation Channel on Investment\*

Job Market Paper

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#### Abstract

This paper evaluates the macro impact of the debt-inflation (Fisher) channel of investment, whereby unexpected inflation erodes the real value of nominal debt and thus stimulates firm-level investment. Consistent with theory, I document new micro evidence that more indebted firms increase investment relative to others following unexpected increase in inflation. To quantify the macro effect of this channel, I develop a general equilibrium model with heterogeneous firms, financial frictions and nominal debt contracts. Calibrated to match key U.S. firm-level moments, the model implies that a 1% unexpected inflation raises aggregate investment by 0.8%. By applying the observed post-COVID inflation surprises, this firm-side debt-inflation is quantitatively important enough to explain about 70% of investment surge. This finding highlights a significant transmission mechanism for investment debt-inflation channel, in contrast to previous studies that found a more modest role for the channel on household consumption.

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#### 1 Introduction

The post-COVID era has been marked by a surge in inflation globally, with rates in the U.S. reaching 40-year highs. This dramatic macroeconomic development has reignited intense academic and policy interest in the real effects of inflation. Among the various transmission mechanisms, the debt-inflation channel, or the Fisher channel—whereby unexpected inflation redistributes wealth from nominal creditors to nominal debtors by revaluing outstanding nominal contracts—stands out as a potentially important force even in low and well-anchored inflation environment. If borrowers have high marginal propensity to consume or invest, this wealth transfer via debt-inflation channel can generate significant real effects on aggregate activities. While the consequences of this channel for household balance sheets and consumption are well-documented (e.g., Doepke and Schneider (2006)), its role on the firm investment has received far less attention. Because non-financial corporations issue a substantial share of nominal debt, the lack of a firm-side perspective limits our comprehensive understanding of the debt-inflation channel.

This paper closes the gap by combining firm-level evidence with a general equilibrium heterogeneous firm model to quantify the macro impact of the debt-inflation channel of investment. In the first part of this paper, guided by a simple conceptual framework, I document the heterogeneous responses using firm-level evidence, which is consistent with the debt-inflation channel on investment. Following an unexpected inflation surprise, more-indebted firms increase investment relative to less-indebted peers. To assess aggregate consequences, I develop a heterogeneous-firm general equilibrium model with nominal debt and financing frictions in the second part of this paper. The calibrated model reproduces the empirical heterogeneity as well as various firm-level moments in the data. At the macro level, in response to a 1% inflation surprise, aggregate investment rises by about 0.83% and output by 0.15%, which are quantitatively sizable effects. During the post-COVID period with inflation surprise, debt-inflation channel alone can explain about 70% of changes in aggregate investment increases. The macro impact of the debt-inflation on investment stands in stark contrast to its typically modest effect on household consumption documented in the existing literature Auclert (2019).

I start by presenting a stylized two-period model as a conceptual framework building on DeAngelo et al. (2011) and Strebulaev et al. (2012), to motivate and guide my empirical specification and the heterogeneity analysis. I analytically demonstrate the debt-inflation mechanism: by devaluing outstanding nominal debt, unexpected inflation relaxes nearly binding financial constraints and in turn raises firm's investment. Two assumptions are central to drive the result. First, firms carry pre-existing nominal debt—both the nominal

rate and the face value of today's repayment were contracted last period; this is the sole source of nominal rigidity. Second, non-negative dividend and borrowing constraints require firms to maintain sufficient internal funds and positive net worth. When the constraint is slack, firms invest up to the unconstrained optimum; when it binds, debt devaluation following unexpected inflation relaxes the constraint and scales up investment. With different endowments of nominal debt, the model shows that more indebted firms are more likely to increase investment in response to an inflation surprise.

I then turn to the data to validate and quantify this mechanism empirically. Using Compustat balance sheet information on debt, I document a key fact: following an increase in unexpected inflation, firms with high indebtedness significantly raise investment relative to those with low indebtedness. I control for firm fixed effects and sector-by-time fixed effects to absorb permanent heterogeneity across firms and differential sector exposure to aggregate changes. Quantitatively, a one-percentage-point increase in inflation is associated with a 35-basis-point relative increase in the investment rate for a firm with one standard deviation higher indebtedness. This finding is robust across various measures of indebtedness and inflation as well as other regression specifications. Using local projection method at longer horizons, the cumulative response peaks around the eighth quarter and then gradually decays. Consistent with the theory, these empirical findings provide the basis for the subsequent general-equilibrium analysis.

While the empirical analysis establishes the existence of the firm-side debt-inflation, it estimates relative responses and cannot identify the full aggregate effect. This is because dynamic adjustments in general equilibrium prices, particularly the real interest rate, are absorbed by the time fixed effects. To recover this "missing intercept" and quantify the channel's aggregate implications, I develop an infinite-horizon heterogeneous firm general equilibrium model with fixed nominal debt contracts and financial frictions. Beyond the two-period illustration used to clarify mechanisms, introducing heterogeneity in productivity and balance sheets allows the model to match the empirical motivation that investment responds differentially by indebtedness and to trace the full cross-sectional dispersion of responses. This design uncovers rich micro heterogeneity that a representative-agent setup would mask and lets me connect panel evidence to aggregate dynamics in a disciplined way. I calibrate the model to match key moments of firms' investment and indebted position in the Compustat data. I then solve the model's transitional dynamics following an inflation surprise using sequence-space Jacobian methods (Auclert et al. (2021)), which efficiently traces the aggregate response to unexpected inflation.

Quantitatively, large inflation surprises operating through the firm-side debt-inflation

can materially shape investment dynamics. In a partial equilibrium setting without endogenous price adjustments, a 1% inflation surprise causes aggregate investment to jump by a substantial 4.5% on impact. This substantial effect demonstrates that the firm-side debt-inflation powerfully relaxes constraints and promotes investment. Once general-equilibrium feedbacks are introduced, especially the rise in the real rate induced by higher investment, the aggregate response attenuates: a 1% inflation surprise increases aggregate investment by 0.83% and output by 0.15%. Relatively highly indebted firms are better off and tend to invest more with the relaxing financial constraint, which pushes up the real rate and in turn hurts other firms. The real interest rate channel works in an opposite direction and depress the investment. Nevertheless, the overall effect on aggregate investment remains positive and significant, highlighting that the debt revaluation mechanism is quantitatively important and distinct from the traditional real interest rate channel. This finding stands in sharp contrast to studies on household consumption, which typically find only minor effects from the debt-inflation.

Delving into the sources of aggregate investment dynamics, I decompose the total effect into level effect and a heterogeneity effect. The level effect captures the wealth transfer from households (net creditors) to firms (net debtors) and it accounts for the majority of the aggregate investment changes. In contrast, the heterogeneity effect, which measures resources flow across heterogeneous firms, is small and slightly negative, modestly dampening the total response. To understand this seemingly counterintuitive result, I further separate within- and between-firm-type effects from the heterogeneity effect. Conditional on firm characteristics, the within-firm-type component is positive: firms with higher debt exhibit stronger investment responses to inflation surprises, precisely as the conceptual framework predicts. However, in the cross-section, the between-firm-type or the compositional effect emerges that overtakes this conditional relationship. High-productivity firms typically maintain larger capital stocks and higher debt levels, but they are also less financially constrained due to stronger cash flows and larger net worth buffers. Consequently, these high-debt firms invest less responsively than their low-productivity, more-constrained counterparts. This negative between-firm-type covariance dominates the positive within-firm-type effect, resulting in the overall heterogeneity effect small and negative. This decomposition highlights that while the debt-inflation channel operates powerfully at the micro level, composition effects in the aggregate distribution can reverse the sign of cross-sectional correlations.

I simulate a panel of firms along the transitional path following inflation shocks and estimate the same regression specifications used in the empirical analysis. The simulated data successfully reproduce the key stylized facts: more indebted firms invest significantly

more than their less-indebted peers following inflation surprises, and the model generates the persistent, hump-shaped dynamic response observed in the local projections. These simulation results confirm that the structural mechanisms embedded in the model can generate the cross-sectional patterns documented in the data. Importantly, since the model reproduces the observed heterogeneous firm responses while being calibrated exclusively to steady state moments rather than targeted to match regression coefficients, this provides confidence that the identified aggregate effect is qualitatively well-captured. This validation underpins the reliability of the counterfactual exercises that follow, where I assess the debt-inflation channel's contribution to major macroeconomic episodes.

Finally, applying the model to actual inflation data reveals that inflation shocks alone can drive non-negligible responses by relaxing financial constraints via debt-inflation, even without other macroeconomic shocks or disruptions. Two episodes are particularly illustrative. During the Great Recession, the large deflationary surprise tightened financial constraints, resulting in a 4% lagged decline in aggregate investment. Compared to actual private non-residential fixed investment, debt-inflation-channel-induced investment dynamics explain approximately 25% of the observed decline. Conversely, during the post-COVID inflation surge, the model predicts a 5% increase in investment relative to its steady-state value. Benchmarking to 2020Q3, the debt-inflation alone can account for up to 70% of the peak increase in investment during this period. These results underscore the firm-side channel's critical importance for capital accumulation, suggesting its macroeconomic significance is substantially more than its counterpart on the household consumption side.

Related Literature This paper mainly contributes to three strands of literature. The first is the literature on the debt-inflation (Fisher) channel, where unexpected inflation redistributes wealth by revaluing nominal contracts, originates with Fisher (1933). This channel gained modern prominence in macroeconomics through Doepke and Schneider (2006), who document its redistributive effects on household balance sheets. Subsequent studies, including Mian et al. (2013), Auclert (2019), Schnorpfeil et al. (2023) and Fagereng et al. (2023), explore implications for consumption and wealth inequality, largely from the household side. Firm-side analysis remain limited. Gomes et al. (2016) examine distorted investment and production decisions in a representative firm setting following unanticipated inflation. Fabiani and Fabio Massimo (2023) suggests positive relationship between stock return and firm's leverage. Brunnermeier et al. (2025) demonstrates how high inflation reduces the real value of corporate debt empirically using German hyperinflation data. None of these papers emphasizes the importance of heterogeneity, as well as its interaction with nominally rigid debt contracts and binding dividend constraints. I argue—empirically and quantitatively—that

heterogeneity is central: I document the micro effect that investment's semi-elasticity to unexpected inflation is higher for more indebted and financially constrained firms, and I develop a heterogeneous-firm general-equilibrium model with nominal debt and financing frictions to quantify the macro impact and to separate debt-devaluation effects. This paper also demonstrates the firm-side debt-inflation is quantitatively meaningful, in sharp contrast to the modest effects on the household side.

Second, this paper adds to the macro-finance literature on investment dynamics under financial frictions. Foundational work shows how balance-sheet frictions amplify shocks (the "financial accelerator") and shape investment dynamics (Bernanke et al. (1999)). Building on this, recent firm-level evidence documents that investment responses to identified monetary shocks vary systematically across firms—by default risk, leverage, and balance-sheet liquidity—and embeds these facts in heterogeneous-firm models (Ottonello and Winberry (2020); Jeenas (2023); Durante et al. (2022); Chen (2023)). My paper introduces a novel interaction into this framework. Rather than frictions merely amplifying fundamental productivity or financial shocks, I show how unexpected inflation directly generates a type of wealth redistribution by devaluing debts, thereby endogenously relaxing frictions that bind firms. This distinguishes our work from Ottonello and Winberry (2020), who study the amplification of monetary policy shocks. By focusing on debt levels (measured as the logarithm of nominal debt) and unexpected inflation, I provide new insights into how financial frictions shape investment responses and a mechanism that is adjacent to (but different from) the conventional interest-rate or credit-spread channels emphasized in the monetary policy literature.

Third, this paper contributes to the literature on real effects with nominal rigidities beyond standard sticky-price and sticky-wage frameworks. While New Keynesian models emphasize frictions in goods and labor markets, a complementary strand highlights non-state-contingent nominal contracts—especially nominal debt—as sources of real effects (e.g., Sheedy (2014); Garriga et al. (2017); Alpanda and Zubairy (2017, 2019)). In contrast, Ippolito et al. (2018) study floating-rate loans, which shows the opposite direction of rigid debt. My contribution is by introducing nominal debt rigidity on the firm side, even with flexible prices, fixed nominal debt contracts interact with financing frictions so that unexpected inflation revalues liabilities and shifts investment, a mechanism orthogonal to and coexisting with standard sticky-price channels. Related ongoing work by Wang and Bai (2025) examines nominal wage contracts as another kind of "liability" for firms, reinforcing the idea that both debt and wage contracts can transmit inflation to real activity through nominally rigid obligations.

**Roadmap** The paper proceeds as follows. Section 2 develops a tractable two-period framework that formalizes the debt-inflation and generates testable predictions. Section 3 takes these predictions to Compustat data, documenting robust heterogeneous investment responses to inflation surprises. Section 4 develops the quantitative heterogeneous-firm model. Section 5 calibrates the model, presents the quantitative results including the aggregate dynamics in response to an inflation shock, and applies the model to two major historical episodes. Section 6 concludes the paper.

# 2 A Conceptual Framework

This section develops a simple two-period model to formalize the debt-inflation channel and derive testable predictions for the empirical analysis. The model shows how unexpected inflation redistributes wealth from creditors to debtors through the revaluation of nominal debt contracts, and how this redistribution translates into heterogeneous investment responses when firms face financial constraints.

## 2.1 Model Description

Environment Consider a model with two periods t = 1, 2, without uncertainty. It simplifies a strand of standard corporate finance model following Hennessy and Whited (2007) and DeAngelo et al. (2011), but there are some specific features added to demonstrate the debt-inflation of inflation. A representative firm produces output using capital with technology  $y_t = k_t^{\alpha}$ , where I assume productivity is normalized to be 1 without any innovations. All prices are exogenously given, including final good prices in two periods,  $P_1, P_2$ , and real interest rate r. The firm begins period 1 with an endowment  $k_1$  and nominal debt burden  $B_1$ , which bears predetermined nominal interest rate  $R_1$ . In practice, nominal bond  $B_t$  is the net value of total bond subtracted by cash holdings, so it can be negative if the firm wants to save rather than borrow. In period 1, knowing full capital depreciation in the next period, the firm chooses its investment level, which determines its capital,  $k_2$ , and issues new one-period nominal debt,  $B_2$ , at a pre-agreed nominal interest rate  $R_2$ .

Following Gomes et al. (2016), define  $b_t = \frac{B_t}{P_{t-1}}$  the real value of debt normalized by the previous period's price level, and let  $\Pi_t = 1 + \pi_t = \frac{P_t}{P_{t-1}}$  be the gross inflation rate. Then real debt value under current price level  $\frac{B_t}{P_t}$  can be reformulated to be  $\frac{b_t}{\Pi_t}$ . Nominal interest rate follows Fisher equation. Given expected inflation is zero (let  $P_2 = P_1$ ), nominal interest rate  $R_t$  is equal to real rate r. The firm chooses the investment  $k_2$  and nominal risk-free corporate debt  $b_2$  to maximize the discounted total value of real dividends  $d_1 = \frac{B_t}{P_t}$ 

 $k_1^{\alpha} - (k_2 - (1 - \delta)k_1) - (1 + r)\frac{b_1}{\Pi_1} + b_2$  and  $d_2 = k_2^{\alpha} + (1 - \delta)k_1 - (1 + r)\frac{b_2}{\Pi_2}$ . It is worth noting that though there is no uncertainty for the second period inflation, the first period inflation is undetermined and subject to change. The firm's objective is to maximize the discounted sum of real dividends,  $d_1$  and  $d_2$ .

**Financial Friction** The model incorporates two crucial financial frictions, which links the firm's net worth to its investment capacity. First, I impose a non-negative dividend constraint

$$d_t > 0$$

This assumption, common in the macro-finance literature, implies that firms cannot issue equity to finance investment and must rely on internal funds (cash flow) or external debt. Usually equity issuance is expensive in terms of both direct and indirect costs and secondly, firms do not issue equity frequently in reality. This constraint is central to our mechanism, as it creates a direct link between a firm's net worth and its investment capacity. Second, to prevent default and ensure the firm can service its debt obligations, I impose a collateral-based borrowing constraint:

$$(1+r)b_2 \le \phi k_2^{\alpha}$$

where  $\phi$  is a parameter governing the tightness of credit markets. This constraint states that nominal repayment on debt cannot exceed a fraction  $\phi$  of the firm's future value. Nominal debt rigidity are embedded in the two frictions here, as it allows unexpected inflation to revalue contracts, unlike indexed debt which neutralizes the Fisher effect.

Overall, the above maximization problem for a firm standing at the beginning of the period 1 can be formally stated as

$$\max_{k_2, b_2} \{ d_1 + \frac{d_2}{1+r} \}$$
s.t. 
$$d_1 = k_1^{\alpha} - k_2 - (1+r) \frac{b_1}{\Pi_1} + b_2 \ge 0$$

$$\phi k_2^{\alpha} - (1+r)b_2 \ge 0$$

Where the second dividend constraint is omitted as the borrowing constraint has a better restriction. Fefine the net worth in the first period

$$nw_1 \equiv k_1^{\alpha} - (1+r)\frac{b_1}{\Pi_1}$$

The two constraints can be combined into a single feasibility requirement:

$$nw_1 \ge k_2 - \phi \frac{k_2^{\alpha}}{1+r}$$

#### 2.2 Optimality and Testable Predictions

The model generates heterogeneous investment responses to inflation surprises based on firms' financial positions.<sup>1</sup>

Unconstrained firms with high net worth  $(nw_1 > k_2^u - \phi \frac{(k_2^u)^\alpha}{1+r})$  invest at the first-best level  $k_2^u = (\frac{\alpha}{1+r})^{\frac{1}{1-\alpha}}$ , independent of inflation surprises. For a given level of initial capital  $k_1$ , they have sufficiently low initial debt have high net worth and invest at first-best level, where the marginal product of capital equals its user cost. For these firms, an unexpected inflation surprise  $\Pi_1 > \mathbb{E}_0\Pi_1$  reduces the real debt burden and increases dividend payout  $d_1$ , but leaves investment unchanged, because they are already at the unconstrained optimum.

Constrained firms with lower net worth face binding constraints, and their optimal investment  $k_2^{\star}$  solves the boundary condition:

$$k_2^{\star} - \phi \frac{(k_2^{\star})^{\alpha}}{1+r} = nw_1$$

For these firms, the debt-inflation operates through net worth: unexpected inflation reduces the real debt burden, raising net worth  $nw_1$ , which in turn relaxes the binding constraint and allows higher investment. Constrained firms with high initial debt  $b_1$  have low net worth  $nw_1$ , making the non-negative dividend constraint binding. These firms are forced to invest below the first-best level  $k_2^* < k_2^u$  because they must divert cash flow to meet the dividend constraint. When unexpected inflation occurs, the real value of debt payment  $\frac{(1+r)b_1}{\Pi_1}$  falls, directly increasing net worth and relaxing the binding constraint. These firms can now increase investment  $k_2^*$ , moving closer to the unconstrained optimum. Consequently, constrained firms with higher initial debt  $b_1$  exhibit stronger investment responses to inflation surprises. This is a testable prediction that forms the basis of my empirical strategy. By the implicit function theorem, the total effect is:

$$\frac{\partial k_2^{\star}}{\partial \Pi_1} = \underbrace{\frac{\partial k_2^{\star}}{\partial nw_1}}_{\text{Marginal Propensity to Invest. Debt. Revaluation}} > 0$$

where the marginal propensity to invest (MPI) out of net worth is  $\frac{1}{1-\frac{\phi\alpha(k_2^*)^{\alpha-1}}{1+r}}$  and debt

<sup>&</sup>lt;sup>1</sup>The detailed derivation of optimality conditions and the proof that  $\phi < \alpha$  ensures non-neutrality of capital structure are provided in Appendix B.

Optimal Investment

Marginal Propensity to Invest

Low Debt
Medium Debt
High Debt
First Best

Figure 1: Optimal Investment and MPI Across Debt Levels

**Notes:** This figure illustrates how firms' optimal second-period investment  $k_2^*$  and its marginal propensity to investment (MPI) out of net worth vary with initial indebtedness  $b_1$ . The left panel shows optimal investment levels as a function of initial debt level  $b_1$  with identical capital stocks  $k_1$ . Black dashed line represents the unconstrained or first-best level of investment. The right panel plots the corresponding MPI  $\frac{\partial k_2^*}{\partial_1}$ . This numerical example is based on the theoretical model with the following core parameters: capital elasticity  $\alpha = 0.8$ , collateral coefficient  $\phi = 0.6$ , and initial capital stock  $k_1 = 1.0$ .

 $b_1$ 

 $b_1$ 

revaluation is  $\frac{(1+r)b_1}{(\Pi_1)^2}$ . Both partial derivatives are strictly positive for constrained firms. The magnitude of this effect depends on two factors: the size of the inflation surprise and the tightness of the financial constraint faced by the firm. The marginal propensity to invest reflect the investment sensitivity to financial conditions, helping understand how much investment respond to an incremental dollar of net worth. It is larger when the borrowing constraint binds more tightly (Lagrangian multiplier is larger, or when  $\frac{\phi \alpha(k_2^*)^{\alpha-1}}{1+r}$  is closer to 1), meaning firms are closer to their borrowing limit and small net worth changes have amplified effects on investment capacity. The second term captures the debt devaluation effect: how much real debt burden falls per percentage point of unexpected inflation. This effect is mechanically larger when initial nominal rates r are higher (more debt service to devalue) and when baseline inflation  $\Pi_1$  is lower (denominator effect). Appendix B provides complete derivations and proves both partial derivatives are strictly positive.

Figure 1 illustrates the core mechanism of the Fisher channel in the two-period model. The left panel shows that optimal investment declines with debt as financial constraints tighten. The right panel demonstrates the key heterogeneity: the marginal propensity to invest (MPI) rises sharply for constrained, high-debt firms. This implies that unexpected in-

flation, which increases firm net worth by eroding real debt, will have differential effects across firms: highly indebted firms will respond more strongly to inflationary shocks, increasing investment more than their less-indebted counterparts. This heterogeneous response forms the theoretical foundation for my empirical identification strategy using firm fixed effects and debt-inflation interactions.

**Empirical Prediction** The two-period model provides clear guidance for empirical testing. Define the investment rate as  $i_1 = \frac{k_2}{k_1}$ . Focusing constrained firms, differentiating the optimality condition with respect to inflation yields:

$$\Delta i_1^{\star} = \Delta \left(\frac{k_2^{\star}}{k_1}\right) = \underbrace{\frac{1}{k_1} \frac{1}{1 - \frac{\phi \alpha (k_2^{\star})^{\alpha - 1}}{1 + r}} \frac{(1 + r)}{(1 + \pi_1)^2}}_{\text{elasticity } \beta} \times b_1 \Delta \Pi_1 \tag{1}$$

The right-hand side of equation (1) takes the form of an interaction term between unexpected inflation  $\Delta\Pi_1$  and initial debt  $b_1$ , which directly motivates the empirical specification in Section 4. This interaction structure is not arbitrary. It emerges naturally from the model's economic mechanism: inflation's effect on investment operates through the revaluation of existing debt, so firms with more debt experience larger balance sheet shocks.

Equation (1) generates several insights about how firm heterogeneity shapes investment responses to inflation surprises. These include predictions about monotonicity in indebtedness (higher  $b_1$  leads to larger responses), constraint interactions (effects concentrate where constraints bind), sorting by productivity (high-MPK firms respond more strongly), and state dependence (aggregate effects depend on the firm distribution). While these predictions are theoretically rich and guide the model's quantitative calibration, my primary empirical focus is on the core prediction that directly tests the existence of the debt-inflation:

The debt-inflation channel hypothesis predicts for constrained firms, the coefficient on the interaction term between initial debt and unexpected inflation should be strictly positive, i.e.  $\beta > 0$ . This prediction follows directly from the debt devaluation mechanism: unexpected inflation reduces the real value of nominal debt obligations, increasing net worth and relaxing financial constraints, which in turn allows higher investment. Testing  $\beta > 0$  is the most direct and robust way to validate the existence of debt-inflation channel. Guided by equation (1), my baseline empirical specification regresses the investment rate on the interaction between unexpected inflation and firms' indebtedness proxy. This specification directly tests whether  $\beta > 0$  in the cross-section, exploiting variation in firms' exposure to the same aggregate inflation shock. The coefficient on the interaction term provides an empirical estimate of  $\beta$ , averaged across constrained firms in the sample. When estimating dynamic effects, I can

estimate local projections to obtain a path  $\{\beta_h\}_{h\geq 0}$ .

Distinguishing from Related Channels Two clarifications help distinguish this channel from related mechanisms in the literature. First, the debt-inflation channel operating through debt revaluation is conceptually distinct from the real interest rate channel emphasized by Auclert (2019). While unexpected inflation lowers the ex-post real rate, the two channels differ fundamentally: the real rate channel affects intertemporal substitution uniformly across agents via expected future rates, whereas the debt-inflation generates cross-sectional redistribution through the revaluation of existing debt stocks on impact. My framework isolates this by holding r constant; in the two-period model, (1+r) enters only through discounting and future borrowing constraints, not through the revaluation term  $(1+r)b_1/\Pi_1$ . While both channels coexist in general equilibrium, I argue the debt-inflation is a first-order effect on impact for constrained firms.

Second, the key nominal rigidity here is distinct from standard New Keynesian frictions. Rather than price or wage stickiness, the mechanism relies on predetermined nominal debt contracts: the nominal rate  $R_t$  is set based on expected inflation  $\mathbb{E}_{t-1}\Pi_t$  via the Fisher equation, but realized inflation deviations create a wedge in the real value of debt. This rigidity is pervasive in practice, corporate bonds, bank loans, and trade credit are overwhelmingly denominated in nominal terms. Yet it has received less attention than goods or labor market frictions. I take this institutional feature as given rather than microfounding it, though costly contract renegotiation or incomplete indexation markets could rationalize the prevalence of nominal debt.

# 2.3 Aggregation and Decomposition

While the two-period model characterizes individual firm behavior, understanding the macroeconomic impact requires aggregating these heterogeneous responses across the firm distribution. This aggregation reveals important insights about the sources of aggregate investment dynamics and the role of firm heterogeneity.

Suppose a firm indexed by i, from the individual firm's optimality condition, the investment response is the product of MPI and debt revaluation  $\frac{\partial k_{i,2}^*}{\partial \Pi_1} = MPI_i \times \frac{(1+r)b_{i,1}}{\Pi_1}$ . Summing across all firms yields the aggregate investment response:

$$\frac{\partial K_2}{\partial \Pi_1} = \sum_{i} \frac{\partial k_{2,i}^*}{\partial \Pi_1} = \underbrace{\mathbb{E}[MPI_i] \times \mathbb{E}\left[\frac{(1+r)b_i}{(1+\Pi_1)^2}\right]}_{\text{Level Effect: household to firms}} + \underbrace{\frac{(1+r)}{(1+\Pi_1)^2} \cdot \text{Cov}(MPI_i, b_i)}_{\text{Heterogeneity Effect: within-firms}}$$
(2)

This decomposition separates the aggregate effect into two components. The first term,

 $\mathbb{E}_i[MPI_i] \times \mathbb{E}_i\left[\frac{(1+r)b_i}{(1+\Pi_1)^2}\right]$  represents the level effect: it captures the redistribution from net creditors to the net debtors<sup>2</sup> weighted by the average marginal propensity to invest across firms. The second term,  $\frac{(1+r)}{(1+\Pi_1)^2} \cdot \text{Cov}_i(MPI_i, b_i)$  represents the heterogeneity effect: it captures whether resources are redistributed toward firms with high or low marginal propensities to invest.

The sign and magnitude of the covariance term determine whether firm heterogeneity amplifies or dampens the aggregate response. If  $\frac{(1+r)}{(1+\Pi_1)^2} \cdot \text{Cov}_i(MPI_i, b_i) > 0$ , meaning firms with more debt also have higher marginal propensities to invest, then heterogeneity amplifies the channel beyond what the mean effect alone would predict. Conversely, if the covariance is negative, heterogeneity partially offsets the mean redistribution effect.

Further Decomposition The two-period analysis in the previous subsection implicitly examines a conditional covariance structure. By holding initial capital  $k_1$  constant and varying debt  $b_1$ , the model isolates the within-firm-type component of the heterogeneity effect. This conditional analysis shows that among firms with the same capital stock, those with higher debt have higher marginal propensities to invest (because they are more likely to be constrained), generating a positive within-group covariance. However, the unconditional covariance aggregated across all firms may differ from this conditional relationship because of composition effects. Suppose we have another state variable for productivity z, the law of total covariance provides a further decomposition:

$$Cov(MPI_i, b_i) = \underbrace{\mathbb{E}_{z,k}[Cov(MPI, b \mid z, k)]}_{\text{Within-firm-type component}} + \underbrace{Cov_{z,k}(\mathbb{E}[MPI \mid z, k], \mathbb{E}[b \mid z, k])}_{\text{Between-firm-type component}}$$
(3)

The first term is the within-firm-type component, averaging the conditional covariance across different firm types (defined by productivity z and capital k). This captures the intuition in the two-period model: holding firm characteristics constant, higher debt leads to tighter constraints and higher marginal propensities to invest. The second term is the between-firm-type component or composition effect. It captures whether high-debt firms systematically differ from low-debt firms along other dimensions that affect investment sensitivity. For instance, if high-productivity firms can borrow more but are also less financially constrained (having larger net worth buffers), they may have lower marginal propensities to invest despite higher debt. This would generate a negative between-group covariance, partially offsetting the positive within-group effect.

<sup>&</sup>lt;sup>2</sup>In Appendix F, I summarize the current sectorial financial positions for the US. Households and firms are now both net creditors while government and external sectors are net debtors. However, compared to household savings, corporate savings are still relatively small. In the framework without government, to have savings equal borrowings, we should have firms to be net debtor.

In summary, the conceptual framework delivers three key insights. First, unexpected inflation acts as a wealth transfer to firms with nominal debt. Second, this transfer translates into higher investment only for financially constrained firms, generating heterogeneous responses based on initial debt positions. Third, the channel's strength depends on the interaction between debt levels and the tightness of financial constraints, yielding a testable prediction:  $\beta > 0$ . The empirical  $\beta > 0$  provides evidence consistent with a positive within-firm-type component in the model. These insights guide both the empirical analysis and the quantitative general equilibrium model developed in subsequent sections.

# 3 Empirical Analysis

In this section, I empirically investigate the heterogeneous response of firm investment rates to unexpected inflation. Our central hypothesis is that the sensitivity of investment to inflation varies systematically with a firm's level of nominal indebtedness.

#### 3.1 Data Descriptions

Inflation Data I obtain monthly Consumer Price Index (CPI) data from the Bureau of Labor Statistics (BLS) and one-year-ahead inflation expectations from the Federal Reserve Bank of Cleveland. The sample period spans from 1990:Q1 to 2023:Q4, encompassing periods of both macroeconomic stability and significant turmoil, including the Great Recession and the post-COVID inflation surge. I aggregate the monthly data to quarterly frequency by averaging within each quarter. The primary measure of unexpected inflation is constructed as the difference between realized quarterly CPI inflation and the one-year-ahead inflation expectation:

$$\varepsilon_t^{\pi} = \pi_t - E_{t-1}\pi_t \tag{4}$$

Several features make this approach particularly suitable for identifying the debt-inflation channel. First, CPI inflation directly reflects movements in the aggregate price level that determine the real value of nominal debt obligations. This contrasts with alternative shock measures based on commodity prices or oil price movements, where the pass-through to the general price level can be slow, incomplete, and heterogeneous across sectors. Since corporate debt contracts are written in nominal terms indexed to the overall price level, CPI-based shocks provide the most direct measure of the wealth transfer from creditors to debtors.

Second, the Cleveland Fed inflation expectations series offers distinct advantages for constructing unexpected inflation shocks. Unlike simple time-series forecasts or backwardlooking measures, the Cleveland Fed estimates incorporate real-time information from financial markets, professional forecasters, and surveys of household expectations, combining these sources through a statistical model that is re-estimated each time new CPI data is released. This ensures that expectations reflect all publicly available information at the time firms make their investment decisions. The series is updated monthly following each CPI release, providing timely and forward-looking measures that better capture how agents actually form expectations in real time.<sup>3</sup>

The resulting unexpected inflation series displays substantial variation over the sample period. Figure 2 presents the time series for realized inflation, expected inflation, and the constructed unexpected inflation shock. As shown in the figure, inflation expectations remained relatively well-anchored throughout most of the sample, particularly during the 2010-2019 period of macroeconomic stability. However, two episodes stand out with significant deviations. During the 2008-2009 Great Recession, deflationary surprises reached approximately negative 2 percentage points as actual inflation fell well below expectations amid collapsing aggregate demand and financial market distress. More recently, the post-COVID period generated large positive inflation surprises, with unexpected inflation exceeding 3 percentage points in 2021-2022 as supply chain disruptions, expansionary fiscal policy, and pent-up demand drove prices up faster than anticipated. <sup>4</sup>

Firm-Level Data I draw firm-level data from the quarterly Compustat database for publicly traded firms in North America. This dataset contains the necessary balance sheet and income statement variables to construct our measures of investment and indebtedness. The sample includes over 268,000 firm-quarter observations spanning 1990:Q1 to 2023:Q4. While the focus on public firms represents a selected sample, these companies hold substantial amounts of corporate debt. According to the Federal Reserve's Financial Accounts of the United States, nonfinancial corporate business debt totaled approximately 72% of GDP as of recent years, and publicly traded firms account for the majority of this debt stock. This makes the Compustat sample economically representative for studying the aggregate implications of the debt-inflation channel. I also draw industry-level capital depreciation data from the Bureau of Economic Analysis (BEA) to help impute investment series.

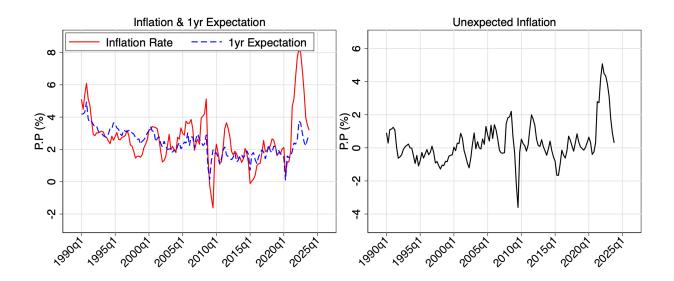
While Compustat firms are typically larger than the average private firm, several considerations support their suitability for studying the debt-inflation channel. A potential concern is that these large public firms may be less financially constrained. Farre-Mensa

<sup>&</sup>lt;sup>3</sup>In Appendix A, I document how to choose timing to address the important measurement issue in details.

<sup>4</sup>The large inflation surprises during crisis periods provide crucial variation that enhances the statistical

The large inflation surprises during crisis periods provide crucial variation that enhances the statistical power of the empirical design, allowing for more precise estimation of the debt-inflation channel's effects. I also conduct robustness checks that exclude the Great Recession and post-COVID periods.

Figure 2: Quarterly Inflation Series (1990Q1-2023Q4)



Notes: The red dashed line represents realized CPI inflation at the quarterly frequency (in percentage points). The blue dashed line shows one-year-ahead expected inflation, aggregated from monthly series. The black solid line depicts unexpected inflation, defined as  $\varepsilon_t^{\pi} \equiv \pi_t^{\text{realized}} - \mathbb{E}_{t-1}[\pi_t]$ . Positive values indicate that realized inflation exceeded expectations from one year prior. This unexpected inflation series is used throughout the empirical analysis. Two episodes stand out: the 2008-09 deflationary shock (reaching -0.9pp) during the financial crisis, and the 2021-22 inflation surprises (peaking at +1.4pp) during the post-COVID recovery. These large deviations provide key identifying variation for the debt-inflation, though results are robust to excluding these periods (see Appendix A).

and Ljungqvist (2016) suggest that public firms which could be identified as "constrained firms" may not be truly constrained because they can access additional borrowing. As emphasized in the previous section, as long as investment opportunity is linked with inflation via net worth, debt-inflation channel can have effects. The institutional features of public firms also actually strengthen the relevance of the channel I study. Considering public firms' size, stability, business diversification, and relationships with external funding partnerships, the likelihood of senior debt default or bankruptcy is considerably low. This motivates my model's focus on risk-free nominal debt rather than incorporating endogenous default, allowing me to isolate the pure debt revaluation mechanism without confounding it with default risk concerns. Moreover, following the pecking order hypothesis Myers and Majluf (1984), public firms exhibit strong reluctance to issue equity due to information asymmetries and signaling concerns. This financing behavior, also broadly modeled in the macro-finance lit-

erature, such as Ottonello and Winberry (2020), makes the non-negative dividend constraint relevant and justifies its role in my theoretical framework.

My main dependent variable is the firm's investment rate  $i_{j,t}$  for firm j at time t, defined as real investment divided by the real capital stock. Following Ottonello and Winberry (2020), I construct real capital series using the perpetual inventory method and then calculate the investment rate at firm level. This approach ensures that the capital stock reflects the actual productive capacity (replacement value) of the firm rather than book values, which can be distorted by accounting conventions and historical cost principles. My primary measure of a firm's financial position is its level of indebtedness  $b_{i,t}$ , measured as the logarithm of total corporate debt (the sum of long-term and short-term debt). To ensure the variable captures firm-specific variation and is comparable across firms and time, I residualize it by removing firm and time fixed effects. This transformation has a clear interpretation: the resulting variable can be seen as the percentage deviation of a firm's debt from its own historical average, purged of aggregate time trends. This ensures that identification comes from withinfirm variation in debt positions over time, interacted with aggregate inflation shocks, rather than from cross-sectional differences in firm size or secular trends in corporate leverage. I prefer this measure over the more conventional leverage ratio for several reasons. First, debtinflation directly operates through the level of nominal debt, which can be directly revalued by unexpected inflation, whereas leverage mixes the debt and asset simultaneously and assets themselves may respond endogenously to inflation through revaluation effects, potentially obscuring the pure debt-inflation channel. The log-deviation interpretation aligns well with the state variable in our theoretical model. Second, leverage ratios in the data exhibit strong and persistent firm fixed effects (Lemmon et al. (2008)), representing stable traits rather than time varying exposure. Third, zero (Strebulaev and Yang (2013)) or aboveone leverage ratios are not rare in Compustat; this accounting fact of negative equity value naturally creates outlier leverage ratios (Jan and Ou (2012)). For robustness concerns, I also present results using the net debt (total debt minus cash equivalents) and leverage ratio (total debt to total assets).

I winsorize all relevant variables at the 0.5% level in both tails to ensure the results will not be driven by outliers. Table 1 provides summary statistics for key variables used in the main results. On average, net real capital grows at 0.4% while investment rate is about 4% per quarter. The difference between the two reflects the capital deprecation. The investment rate exhibits substantial variation, with a standard deviation of 10.3 percentage points and a 95th percentile of 15%, reflecting considerable heterogeneity in firm investment dynamics. The main explanatory variable, log debt  $b_{j,t-1}$ , has a mean of 4.0 and a standard deviation

Table 1: Descriptive Statistics for Firm Investment and Indebtedness

Statistic	$\Delta \log k_{j,t}$	$i_{j,t}$	$b_{j,t-1}$
Mean	0.362	3.936	3.984
Median	-0.443	2.723	4.149
S.D.	8.729	10.263	2.993
95th Percentile	11.182	14.997	8.520
Observations	268757	268757	268757

**Notes:**  $k_{j,t}$  is the real capital computed using perpetual inventory method from balance sheet information and industry average depreciation data from BEA following Ottonello and Winberry (2020).  $\Delta \log k_{j,t} \equiv \log k_{j,t} - \log k_{j,t-1}$  is the net real capital change.  $i_{j,t}$  is calculated as the real investment divided by capital stock in last period. Both series are in the unit of percentage points.  $b_{j,t-1}$  is the logarithm of total debt dlcq + dlttq from Compustat.

of approximately 3 log points. This metric facilitates economic interpretation of regression results when comparing firms with different indebtedness levels. In Appendix A, I also present sample statistics using the leverage ratio, where the sample size is larger than the log debt sample because firms with negative or zero debt are dropped due to the logarithm transformation.

#### 3.2 Empirical Strategy

The goal of this empirical exercise is to identify and quantify the debt-inflation channel by linking firm's investment to the interaction between unexpected inflation and its indebtedness position. In the era of high inflation, creditors generally lose in real value but debtors win. Firms with higher nominal indebted positions should have less real financial burden relative to their less indebted peers, leading to relatively improved cash flow and net worth. The core hypothesis is that for firms with significant nominal debt, an unexpected inflation acts as a positive wealth shock, relaxing financial constraints and stimulating investment.

**Baseline Specification** To test this, I estimate the following regression specification, directly motivated by the theoretical relationship in equation (1):

$$i_{j,t} = \alpha_j + \alpha_{s,t} + \beta(x_{j,t-1} \times \varepsilon_t^{\pi}) + \gamma x_{j,t-1} + \Gamma_A'(x_{j,t-1} \times \mathbf{A}_t) + \Gamma_Z' \mathbf{Z}_{j,t-1} + e_{j,t}$$
 (5)

where  $i_{j,t}$  is the firm j's real investment rate in quarter t. Firm fixed effects  $\alpha_j$  absorb time-invariant heterogeneity. Sector-by-time fixed effects  $\alpha_{s,t}$  absorb all aggregate shocks common to firms within an industry-quarter, including the direct effect of unexpected inflation  $\varepsilon_t^{\pi}$ , other business cycle dynamics, sector-specific demand shifts and so on.  $x_{j,t-1}$  is the proxy

for firm j's indebtedness at the end of period t-1, and  $\epsilon_t^{\pi}$  is the unexpected inflation at time t. In the main context, it is the standardized logarithm of residualized total debt  $b_{j,t-1}$ . Following Ottonello and Winberry (2020), the transformation means the estimate is driven by how a given firm responds to unexpected inflation when it has higher indebtedness than its own historical average, rather than by comparing permanently high-debt firms to permanently low-debt firms. The key explanatory variable is therefore the interaction term  $(x_{j,t-1} \times \varepsilon_t^{\pi})$ .

In order to isolate the debt-inflation channel effect from other mechanisms through which inflation or monetary policy might differentially affect firms, I include interaction terms between the financial proxy  $x_{j,t-1}$  and a vector of aggregate controls  $\mathbf{A}_t$ , which consists of real GDP growth and the federal funds rate. These controls address two potential confounds. First, unexpected inflation often correlates with output growth, and highly leveraged firms may be more sensitive to demand shocks through credit constraints. Second, monetary policy responses to inflation (changes in the federal funds rate) can affect investment through the user cost of capital and borrowing constraints. By including these interaction terms, I ensure that  $\beta$  isolates the pure debt-inflation effect, holding constant differential sensitivities to growth and interest rate changes. The specification also includes a vector of standard firm-level controls  $Z_{j,t-1}$ , widely used in the corporate finance and investment literature. It includes total assets, sales growth, current asset ratio and Tobin's Q measure.

This specification exploits a difference-in-differences logic with continuous treatment. It effectively compares the investment response of two firms in the same sector and quarter but differing in their indebtedness exposure. Consider a concrete example: two manufacturing firms in 2021Q4, both facing a 1 percentage point inflation surprise. If one firm has 1 standard deviation higher log debt than another, the specification predicts this firm invests  $\beta$  percentage points more. Critically, the sector-time fixed effects absorb the direct level effect of unexpected inflation on investment, as well as other business cycle dynamics and sector-specific demand shifts. This implies that the specification cannot identify how much aggregate investment responds to inflation in absolute terms, only how the response varies across firms with different debt positions, which is known as the "Missing Intercept" problem.<sup>5</sup> Recovering the aggregate effect requires general equilibrium reasoning, which motivates the structural model in Section 4.

The key estimate of interest is  $\beta$ , which measures how investment responds to unexpected inflation conditional on indebtedness. The debt-inflation channel predicts that  $\beta$  should

<sup>&</sup>lt;sup>5</sup>To illustrate: if unexpected inflation boosts investment uniformly across all firms by 0.5%, this common effect is absorbed by  $\alpha_{s,t}$ , and only the differential response (e.g., high-debt firms invest 0.3% more than low-debt firms) is identified through  $\beta$ 

be positive: a positive unexpected inflation ( $\epsilon_t^{\pi} > 0$ ) should have a more positive (or less negative) effect on investment for firms with higher relative indebtedness than usual  $(x_{j,t-1})$ . Economically,  $\beta$  captures the marginal propensity to invest out of the wealth transfer induced by debt revaluation. The coefficient  $\gamma$  captures the direct effect of indebtedness on investment. Consistent with standard corporate finance and investment literature, I expect  $\gamma$  to be negative, as a higher debt burden generally depresses investment. It is crucial to emphasize that the debt-inflation channel does not imply that debt promotes investment in general. Rather, it suggests that unexpected inflation mitigates the negative effect of debt on investment through balance sheet relief. In other words, high-debt firms may still invest less than low-debt firms on average (captured by  $\gamma < 0$ ), but they respond more strongly to inflation surprises (captured by  $\beta > 0$ ). Due to the limited coverage of firms in Compustat, the regression estimate should be regarded as a lower bound of the true investment responsiveness, because publicly traded firms are often larger but less financially constrained than small private firms.

To trace the dynamic effects of the channel, I employ the local projection method of Jordà (2005). I estimate the following equation for horizons h=0,1,...,12 quarters:

$$\Delta \log k_{j,t+h} = \alpha_j + \alpha_{s,t} + \beta_h(x_{j,t-1} \times \varepsilon_t^{\pi}) + \gamma_h b_{j,t-1} + \Gamma'_{Ah}(x_{j,t-1} \times \mathbf{A}_t) + \Gamma'_{Zh} \mathbf{Z}_{j,t-1} + e_{j,t,h}$$
 (6)

where  $\Delta \log k_{j,t+h} = \log k_{j,t+h} - \log k_{j,t-1}$ , represents the cumulative log change in the capital stock over  $h \geq 0$  periods. This measure captures the net capital accumulation over hh h periods, accounting for both new investment and depreciation. It provides a more comprehensive view of how the debt-inflation channel affects firms' capital trajectories over time. The coefficient  $\beta_h$  traces out the dynamic impact of the interaction between debt and unexpected inflation on capital accumulation at horizon h.

# 3.3 Main Empirical Results

Table 2 presents the main regression results, using the standardized log of debt as the measure of indebtedness. All specifications include firm fixed effects and sector-by-quarter fixed effects, as well as interactions of indebtedness with GDP growth and the federal funds rate to isolate the debt-inflation channel from confounding business cycle effects. Standard errors are clustered at both the firm and time level. Column (1) shows shows the baseline specification using unexpected inflation, while column (2) is the preferred specification with firm level controls. Column (3) and (4) present corresponding results using realized inflation rather than unexpected inflation. All macro variables including the direct effects of inflation

shocks are absorbed by the sector-time fixed effects and therefore are not reported in the table.

All four columns consistently show that firms with higher indebtedness position are more responsive to inflation surprises. Column (2), my preferred specification, yields a coefficient of 0.124 (standard error 0.029) on the interaction between unexpected inflation and indebtedness, statistically significant at the 1% level. The stability of the coefficient across columns (1) and (2) suggests that the interaction effect is not driven by omitted firm characteristics. For a one-percentage-point increase in unexpected inflation, a firm with one standard deviation higher indebtedness (2.99 log points) increases its investment rate by an additional 0.37 percentage points ( $0.124 \times 2.99$ ). This represents approximately 9.4% of the sample mean investment rate of 3.94 percentage points. This substantial differential response provides strong support for the debt-inflation. Columns (3) and (4) using realized inflation show smaller but still significant coefficients of 0.089 and 0.091, respectively, as expected since realized inflation conflates anticipated and unanticipated components.

The coefficients on indebtedness itself are negative, highly significant and consistent with the theory that more indebted firms have lower on average investment. The negative estimate of interaction with GDP growth suggests that during economic expansions, more indebted firms benefit relatively less, highlighting that the inflation channel operates distinctly from an aggregate demand channel. Similarly, the negative coefficient on the federal funds rate interaction confirms that tighter monetary policy disproportionately harms more indebted firms, consistent with the financial accelerator mechanism.

Figure 3 plots the dynamic impulse responses from the local projections (Equation 6). The dependent variable is the cumulative log change in real capital stock from period t-1 to period t+h, tracing how an inflation shock affects the capital trajectory over the subsequent three years. The results reveal substantial and highly persistent effects. The response is positive and statistically significant at all horizons from h=0 to h=12. On impact, the coefficient is approximately 0.11, close to the baseline regression coefficient in Table 2. The cumulative response reaches its peak around the eighth quarter at approximately 1, then gradually decays but remains significantly positive even after three years. The upward-sloping path over the first two years implies that the investment rate remains elevated for several quarters following the initial shock. This persistence indicates that the debt-inflation channel is not merely a transitory liquidity effect. Several mechanisms could explain this: convex adjustment costs lead firms to spread investment over time, improved balance sheets provide sustained access to external finance, and investment projects could have lags. The hump-shaped response suggests that the channel has meaningful medium-run implications

Table 2: Main Results

Investment Rate	(1)	(2)	(3)	(4)
Unexp. Inflation $\times$ Indebtedness	0.116***	0.124***		
	(0.029)	(0.029)		
Inflation $\times$ Indebtedness			$0.089^{***}$	$0.091^{***}$
			(0.023)	(0.023)
Indebtedness	-0.547***	-0.414***	-0.681***	-0.549***
	(0.075)	(0.082)	(0.078)	(0.085)
GDP Growth $\times$ Indebtedness	-0.044**	-0.045**	-0.046**	-0.047**
	(0.018)	(0.018)	(0.018)	(0.018)
$FFR \times Indebtedness$	-0.054***	-0.060***	-0.078***	-0.085***
	(0.020)	(0.020)	(0.022)	(0.022)
Observations	268757	268757	268757	268757
$R^2$	0.118	0.125	0.118	0.124
Firm Control	No	Yes	No	Yes
Time Sector FE	Yes	Yes	Yes	Yes
Two-way Clustering	Yes	Yes	Yes	Yes

Notes: Standard errors in parentheses. \*p < 0.10, \*\*p < 0.05, \*\*\*p < 0.01. Firm controls as indicated; all specifications include time-by-sector fixed effects; standard errors clustered by time and firm. Results are estimated from regression specification 5. First two columns employ unexpected inflation series  $\varepsilon_t^{\pi}$  constructed above, while third and fourth column show results directly using CPI inflation.  $\mathbf{Z_{j,t}}$  is a vector of control variables at firm level and interaction of demeaned financial position with GDP growth and federal funds rate. Estimates of unexpected inflation or inflation series are absorbed by time fixed effects.

for capital accumulation.

#### 3.4 Robustness Check

In Appendix A, I report a comprehensive set of robustness checks that validate the baseline empirical findings and further isolate the debt-inflation effects on firm investment responses to unexpected inflation. These exercises address potential concerns about sample composition, variable construction, and alternative transmission mechanisms, ensuring the results are not artifacts of specific assumptions or data peculiarities.

First, to distinguish the debt channel from confounding effects through other firm characteristics, I augment the baseline specification with interaction terms between unexpected inflation and a variety of firm-level variables. These augmented specifications test whether the debt interaction remains significant after controlling for alternative channels through which inflation might differentially affect firms. the additional interactions include sales

1.6 1.4 Interaction coefficient 1.2 1 8.0 0.6 0.4 0.2 0 -0.20 1 2 3 5 6 8 9 4 7 10 11 12

Figure 3: Dynamic Impacts after the Inflation Surprise

**Notes:** Plots the estimated coefficient  $\beta_h$  over h quarters following local projection regression (3). Dashed lines report 90% error bonds. Estimation of cumulative capital change is different from the investment rate from on impact regression.

Quarters since shock

(to capture potential sales inflation effects or the cash flow channel documented in Bhamra et al. (2023)), liquidity (Jeenas (2019)), firm size (scale dependent responses) and firm age (life-cycle dependent responses), and dividend payouts (for payout policy influences). Across these specifications, the core interaction estimate of interest remains positive, statistically significant, and quantitatively stable relative to the baseline estimate. Notably, including interactions with sales or firm size does not attenuate the debt interaction coefficient, underscoring that the estimated differential response arises specifically from debt devaluation, where inflation erodes real debt burden, rather than from nominal revenue boosts, scale economies, or other correlated channels that might disproportionately affect larger or faster-growing firms.

Second, the results are robust to alternative measures of firm indebtedness. While my baseline specification uses the log of gross debt, one might be concerned that this measure includes short-term operational debt or fails to account for liquid assets that firms could use to repay debt. I re-estimate the baseline specification replacing gross debt with: (i) net debt (total debt minus cash and short-term investments), which captures the firm's net obligation to creditors; (ii) long-term debt only, which focuses on multi-year obligations more clearly subject to inflation erosion; and (iii) the leverage ratio (total debt to total assets), the con-

ventional measure in corporate finance. All alternative measures yield consistently positive and significant interaction coefficients. The smaller coefficient for leverage is expected, as this measure conflates debt with asset values, both of which may respond to inflation, potentially diluting the measured effect. The consistency across debt measures confirms that the channel operates through nominal debt exposure rather than being an artifact of a particular accounting definition.

Third, subsample analyses rule out undue influence from extreme macroeconomic periods. I re-estimate the baseline specification on several restricted samples: (i) post-1994 data only, which excludes the early 1990s recession; (ii) excluding both the Great Recession and post COVID periods jointly; and (iii) dropping each crisis separately. The debt-inflation interaction remains positive and significant across all subsamples, with coefficients ranging from 0.076 to 0.127. This demonstrates that while the extreme episodes provide valuable statistical power, the debt-inflation channel operates consistently during normal times as well.

Taken together, these robustness checks affirm that the debt-devaluation channel is a robust, persistent feature of the data, driving differential investment responses to inflation surprises independently of alternative mechanisms, sample choices, or debt metrics. The heterogeneous firm responses documented in this section provide strong micro-level evidence for the existence and economic significance of the channel. Detailed tables and figures, including graphical illustrations of interaction effects and subsample comparisons, are provided in the Appendix A. To back from the relative differential effects to the aggregate level effects, in what is to follow, I develop a model which nests the debt-inflation channel in a macroeconomic framework.

As emphasized earlier, the panel regression approach with sector-time fixed effects cannot identify the aggregate effect of inflation on investment due to the Missing Intercept problem. The estimates capture only the slope: how the response varies across firms with different debt levels, but not the overall level effect. To quantify the macroeconomic implications of the debt-inflation channel and recover the aggregate investment response, I now turn to a structural general equilibrium model that nests the micro-level mechanism within a complete macroeconomic framework.

# 4 Quantitative Model of the debt-inflation

In this section, I extend the two-period model to a quantitative general equilibrium model with firm heterogeneity, fixed nominal debt and financial frictions. The model preserves the

core mechanism from Section 3 while embedding it in a richer environment that allows for meaningful quantitative analysis. The key extensions include: (i) persistent idiosyncratic productivity shocks that generate endogenous heterogeneity in firm size and financial positions, (ii) convex capital adjustment costs that create realistic investment dynamics, (iii) exogenous entry and exit to maintain a stationary distribution, and (iv) a complete specification of general equilibrium with households, retailers, and market clearing conditions.

#### 4.1 Heterogeneous Firm

Wholesale Firms There is a continuum of firms on measure (0, 1) indexed by i. Each firm produces using a decreasing returns to scale technology:

$$y_{i,t} = z_{i,t} k_{i,t}^{\alpha} n_{i,t}^{\nu}, \quad \alpha + \nu < 1$$
 (7)

where z is a firm-specific productivity, which follows an AR(1) process in logs

$$\log(z_{i,t+1}) = \rho \log(z_{i,t}) + \sigma \varepsilon_{i,t+1}, \quad \varepsilon_{j,t+1} \sim N(0,1)$$
(8)

and  $k_{i,t}$ ,  $n_{i,t}$  are capital and labor used to produce wholesale goods. Firms make investment  $i_{i,t}$  to accumulate capital

$$k_{i,t+1} = i_{i,t} + (1 - \delta)k_{i,t} \tag{9}$$

subject to convex adjustment costs,

$$AC(i_{i,t}, k_{i,t}) = \frac{\gamma}{2} \frac{i_{i,t}^2}{k_{i,t}} = \frac{\gamma}{2} \frac{(k_{i,t+1} - (1-\delta)k_{i,t})^2}{k_{i,t}}$$
(10)

The quadratic adjustment cost specification, common in the investment literature, is important for matching the persistent investment dynamics observed in the data.

I retain the two core financial frictions from the two-period model. First, firms face a non-negative dividend constraint, which restricts them from issuing new equity and makes debt the sole source of external financing. Second, firms can issue one-period, risk-free nominal corporate bonds,  $B_{i,t+1}$ . The key assumption is that debt contracts are written in nominal terms with the nominal interest rate  $R_{t+1}^{6}$  predetermined based on expected inflation according to Fisher equation. This creates the nominal rigidity necessary for the debt-inflation channel to operate: unexpected deviations in realized inflation revalue the real

<sup>&</sup>lt;sup>6</sup>Central bank does not directly set nominal interest based on Taylor rule. In this simplified model, it only supplies money to meet money demand and to clear money market.

debt burden ex post. The net debt position,  $B_{i,t+1}$ , can be negative, which I interpret as cash holdings or savings<sup>7</sup>. While traditional models often introduce frictions like tax shields to incentivize borrowing, I allow for saving to reflect the empirical fact that many firms hold substantial cash for precautionary or strategic reasons, as documented in the data.

These wholesale firms sell their outputs in a competitive market at real relative price  $p_t$ . The real dividend for a continuing firm is given by:

$$d_{i,t} = p_t z_{i,t} k_{i,t}^{\alpha} n_{i,t}^{\nu} - w_t n_{i,t} - i_{i,t} - AC(i_{i,t}, k_{i,t}) - (1 + R_t) \frac{b_t}{\Pi_t} + b_{i,t+1}$$
(11)

where  $b_{i,t} = \frac{B_{i,t}}{P_{t-1}}$  and  $b_{i,t+1} = \frac{B_{i,t+1}}{P_t}$ . To ensure debt is risk-free and prevent default, the debt repayment must be collateralized by the firm's value in the worst possible future state. This implies a borrowing constraint:

$$(1 + R_{t+1})b_{i,t+1} \le \prod_{t+1} \phi(p_{t+1}\underline{z}_{i,t+1}k_{i,t+1}^{\alpha}n_{i,t+1}^{\nu} - w_{t+1}n_{t+1} + (1 - \delta)k_{i,t+1})$$

$$(12)$$

where  $\underline{z}$  is the lowest value of realized productivity and  $\phi$  is a parameter measuring credit market tightness. In the baseline calibration, I set  $\phi = 1$ , meaning firms can borrow up to the expected value of their assets in the worst state. Lower values of  $\phi$  would represent tighter credit conditions.

Finally, To maintain a stationary distribution without modeling endogenous default, I assume an exogenous exit probability  $\pi_d$ . At the beginning of each period, each firm draws an i.i.d. death shock. Exiting firms liquidate their assets to pay off debt after production, without making new investment or borrowing decisions. New firms enter debt-free with an initial capital endowment  $k_0$ . This entry-exit structure serves two purposes: it prevents successful firms from growing without bound (which would occur under decreasing returns without exit), and it provides a simple way to match the observed churning in the firm distribution without introducing the computational complexity of endogenous default.

The firm's problem is to choose investment and debt based on the state z, k, b to maximize

<sup>&</sup>lt;sup>7</sup>I do not feature any other risk free asset such as government bond, but it can be easily added to accommodate savings. It can be thought of that firms are saving to meet the demand for household borrowing.

its value V , which is a weighted average of its continuation value  $V_c$  and exiting value  $V_d$ :

$$V_{t}(z, k, b) = (1 - \pi_{d}) V_{t}^{c}(z, k, b) + \pi_{d} V_{t}^{d}(z, k, b)$$

$$V_{t}^{c}(z, k, b) = \max_{k', b'} \left\{ d(z, k, b, k', b') + \mathbb{E} \left[ \Lambda' V_{t}(z', k', b' \mid z) \right] \right\}$$

$$s.t. \qquad d_{t} = p_{t} z k^{\alpha} n^{\nu} - w_{t} n - i - AC(i, k) - (1 + R_{t}) \frac{b}{\Pi_{t}} + b' \ge 0$$

$$b' \le \frac{\Pi_{t+1}}{1 + R_{t+1}} (p_{t+1} \underline{z}' k'^{\alpha} n'^{\nu} - w_{t+1} n' + (1 - \delta) k')$$

$$(13)$$

where  $\Lambda$  is the stochastic discount factor from the household problem (defined below), and the bellman equation is subject to the non-negative dividend and borrowing constraints.

Appendix C provides the complete derivation of the first-order conditions using the Lagrangian method. Here I summarize that the firm's investment Euler condition that pins down contemporaneous investment can be written as

$$(1+\xi_t)\Big(1+\gamma\frac{i_t}{k_t}\Big) = \mathbb{E}_t\big[\Lambda_{t,t+1}V_{k'}(z',k',b')\big] + \psi_t \frac{\partial \mathcal{B}_t(k')}{\partial k'},$$

where  $\xi_t \geq 0$  is the multiplier on the dividend non-negativity constraint and  $\psi_t \geq 0$  is the multiplier on the collateral-based borrowing constraint. The left-hand side represents the marginal cost of investment, which includes both the direct cost of an additional unit of capital (inclusive of adjustment costs) and the shadow cost from the dividend constraint when it binds ( $\xi_t > 0$ ). The right-hand side represents the marginal benefit, consisting of the expected marginal value of capital plus the benefit of relaxing the collateral constraint. The debt Euler takes the form

$$1 = \mathbb{E}_t \left[ \Lambda_{t,t+1} \, \frac{1 + R_{t+1}}{\prod_{t+1}} (1 + \xi_{t+1}) \right] + \psi_t.$$

This condition equates the marginal cost of borrowing today (left-hand side) to the marginal benefit, which includes both the expected discounted cost of repayment (first term on the right) and the benefit of relaxing the current borrowing constraint (second term).

These two conditions make clear the channels through which inflation affects investment. A binding dividend multiplier ( $\xi_t > 0$ ) amplifies the effective marginal cost of investment on the left hand side and thus dampens immediate investment responses. The borrowing multiplier  $\psi_t$  enters both conditions and captures how credit tightness constrains optimal choices. Appendix C proves that, under standard regularity assumptions and for firms initially constrained, a small unexpected increase in gross inflation  $\Pi_t$  reduces both  $\psi_t$  and  $\xi_t$  locally, which in turn raises contemporaneous investment.

**Timing:** At the beginning of the period t, each firm's idiosyncratic productivity,  $z_{i,t}$  realizes. Then, an i.i.d. "death shock" determines whether the firm will exit at the end of the period. Firms designated to continue will produce, invest, pay off old debt, and issue new bonds. Firms designated to exit will only produce and pay off their debt before leaving the market. To keep the total mass of firms constant at one, a measure  $\pi_d$  of new firms enters at the end of each period. These entrants start with initial capital  $k_0$  and zero debt, assuming their initial funding comes from equity provided by households. The evolution of the cross-sectional distribution of firms,  $\mu_t$ , is thus governed by:

$$\mu_{t+1}(z', k', b') = \int (1 - \pi_d) \mathbf{1}\{k' = k^*(z, k, b)\} \mathbf{1}\{b' = b^*(z, k, b)\} g(z' \mid z) d\mu_t(z, k, b)$$

$$+ m_{\text{ent}} \mu_{\text{ent}}(z') \mathbf{1}\{k' = k_0\} \mathbf{1}\{b' = 0\},$$
(14)

where  $k^*(z, k, b)$  and  $b^*(z, k, b)$  are the optimal policy functions, g(z'|z) is the transition density for productivity, and  $\mu_{\text{ent}}(z')$  is the initial productivity distribution for entrants.

#### 4.2 Other Agents and Market Clearing

To close the model, I introduce three other standard agents.

Retailers and Final Goods Producer A continuum of monopolistically competitive retailers indexed by j buy wholesale goods and differentiate them using a linear technology  $\tilde{y}_j = y$ . A final goods producer then aggregates these differentiated varieties using a constant elasticity of substitution (CES) technology,  $Y_t = \left(\int_0^1 \tilde{y}_{jt}^{\frac{\epsilon_p-1}{\epsilon_p}} dj\right)^{\frac{\epsilon_p}{\epsilon_p-1}}$  with standard price index  $P_t = \left(\int_0^1 \tilde{P}_{jt}^{1-\epsilon_p} dj\right)^{\frac{1}{1-\epsilon_p}}$  where  $\tilde{P}_j$  is the price for differentiated goods j. This setup yields a constant markup, such that the real price of wholesale goods is  $p = \frac{\epsilon_p-1}{\epsilon_p}$  in steady state. The introduction of retailers creates the wedge between wholesale and retail prices, but does

**Households** A representative household maximizes expected lifetime utility from consumption and labor:

not affect the debt-inflation channel, which operates through wholesale firms' balance sheets.

$$U(\{C_t\}, \{N_t\}) = E_0 \sum_{t=0}^{\infty} \beta^t (\log C_t - \chi N_t)$$
(15)

<sup>&</sup>lt;sup>8</sup>In the model, I keep flexible prices to fully isolate the debt-inflation effect, which is the main focus of this paper. But by adding normal rigidities, this model can be extended to study monetary policy designs in the future.

subject to the budget constraint

$$P_t C_t + S_{t+1} = W_t N_t + (1 + R_t) S_t + D_t$$
(16)

where  $S_t$  represents nominal savings in the form of corporate bonds,  $W_t$  is the nominal wage, and  $D_t$  is the aggregate dividend received from the corporate sector. The household's optimization provides the economy's stochastic discount factor

$$\Lambda_{t,t+1} = \beta \frac{u'(C_{t+1})}{u'(C_t)} = \beta \frac{C_t}{C_{t+1}}$$
(17)

The linear disutility from labor,  $\chi N_t$ , implies a perfectly elastic labor supply, where the real wage  $w_t$  is pinned down by the marginal rate of substitution.

Market Clearing The goods market clears when total production equals the sum of consumption, investment, and adjustment costs, accounting for entry and exit:

$$\int y_{jt}d\mu_t = Y_t = C_t + (1 - \pi_d) \int (i_{jt} + AC_{jt})d\mu_t + \mu_{ent}k_0 - \pi_d(1 - \delta)K_t$$
 (18)

Since corporate bonds is the only asset in the market, the asset market clears implicitly, as household savings must equal the aggregate net debt of the corporate sector:

$$\int b_{i,t}d\mu_t = \frac{S_t}{P_{t-1}} \tag{19}$$

Labor market clears following labor supply meets labor demand

$$\int n_{i,t}d\mu_t = N_t \tag{20}$$

**Equilibrium** The steady state equilibrium for the flexible price economy is given by a set of value functions  $V_t(z_{i,t}, k_{i,t}, b_{i,t})$ , decision rules  $k_{i,t+1}, b_{i,t+1}, n_{i,t}$  for capital, debt and labor, a measure of firms  $\mu_t(z_t, k_t, b_t)$ , and a set of prices  $w_t, r_t, p_t, \Lambda_{t,t+1}$  such that: (i) given prices, all firms optimize: V solves bellman equation with associated policy rules; (ii) household optimizes; (iii) goods market, labor market and asset market all clear; and (iv) the distribution of firms  $\mu$  is stationary.

# 5 Unexpected inflation and Quantitative Analysis

In this section, I quantify the macroeconomic effects of unexpected inflation through the debt-inflation channel. The flexible-price environment provides a clean laboratory for isolating this mechanism. By introducing an exogenous unexpected inflation surprise, driven by a one-time change in the money supply, I can trace out its real effects purely through debt devaluation.

#### 5.1 Calibration

I calibrate the model at a quarterly frequency to match key features of the U.S. economy and firm-level data from Compustat. Following the standard approach in the quantitative macroeconomics literature, all parameters are set externally based on established values from prior studies and empirical estimates. This parsimonious calibration strategy allows me to evaluate the model's ability to match key moments in the data without overfitting through targeted parameter selection. Table 3 presents my calibration strategy.

The household discount factor  $\beta$  is set to 0.99, implying an annual real interest rate of approximately 4% in steady state. The quarterly depreciation rate  $\delta$  is set to 0.025, corresponding to an annual depreciation rate of 10%, consistent with values commonly used in business cycle models and BEA data.

For the production technology, the capital share  $\alpha$  is set to 0.25 and the labor share  $\nu$  is set to 0.60, such that  $\alpha + \nu = 0.85$ . This calibration generally follows Ottonello and Winberry (2020) and ensures decreasing returns to scale at the firm level. The combined returns to scale parameter also helps the model generate aggregate employment shares consistent with the data, with roughly  $\frac{\epsilon_p - 1}{\epsilon_p} \nu \approx 55\%$  of output accruing to labor, which is close to the current aggregate labor share around 59%. The idiosyncratic productivity process is calibrated based on estimates from the firm dynamics literature. I set the persistence parameter  $\rho_z$  to 0.90 and the standard deviation of innovations  $\sigma_z$  to 0.10, values within the ranges estimated for U.S. firms (Ottonello and Winberry (2020), Catherine et al. (2022)). I discretize the continuous AR(1) into 7 states using the Tauchen (1986) method as implemented by Terry and Knotek II (2011).

The parameters governing financial frictions are chosen based on empirical evidence and prior literature. The borrowing constraint parameter  $\phi$  is set to 1.00, reflecting the typical collateral value of firm assets and implying that firms can borrow up to the full present value of their productive assets. The exogenous exit probability  $\pi_d$  is set to 0.02 per quarter, corresponding to an annual exit rate of approximately 8%, consistent with empirical evidence on firm turnover in the United States. The investment adjustment cost parameter  $\gamma$  is set to 1.00, within the range of 0.045 to 3 commonly used in the investment literature (Cooper and Haltiwanger (2006) and Winberry (2021)). This choice helps generate realistic investment dynamics while avoiding computational difficulties that arise with either very small or very

Table 3: Calibrated Parameters for the full model

Description	Parameter	Value	Source/Target	
Household				
Discount factor	$\beta$	0.99	Fixed	
Labor Disutility	$\chi$	1.07	60% Employment Rate	
Firm				
TFP Persistence	$ ho_z$	0.9	Ottonello and Winberry (2020)	
SD of Innovations to TFP	$\sigma_z$	0.1	Catherine et al. (2022)	
Depreciation Rate	$\delta$	0.025	10% Annual Depreciation (BEA)	
Coefficient on Capital	$\alpha$	0.25	Return to Scale = $85\%$	
Coefficient on Labor	$\nu$	0.6	SS Labor Share $\approx 54\%$	
Borrowing Limit	$\phi$	1	Gross Leverage	
Exogenous Death Rate	$\pi_d$	0.02	8% Annual Exit	
Investment Adjustment Cost	$\gamma$	1	0.05 - 2	
Initial Capital for Entrants	$k_0$	0.2	Employment Size Ratio <sup>a</sup>	
Elasticity of Substitution	$\epsilon_p$	10	Ottonello and Winberry (2020)	

<sup>&</sup>lt;sup>a</sup> Source: U.S. Census Bureau – CES, Business Dynamics Statistics (2022), the ratio is employment of firms < 1 year old divided by firms > 10 years old,  $\approx 0.0216$ .

large adjustment costs. The initial capital endowment for entrants  $k_0$  is set to 0.20 to match the young and old firm size ratio from Census Bureau's Business Dynamics Statistics (BDS).

Computational Implementation I solve for the steady-state equilibrium using value function iteration on a discrete state space. The state space consists of 7 productivity states, 120 capital grid points and 100 debt grid points. When computing for distribution, I interpolate policy functions with denser 200 capital and 150 debt grid points and use nonstochastic simulation method in Young (2010).

Model Fit Despite relying entirely on externally calibrated parameters without any targeted moment matching, the model performs remarkably well in fitting key features of the firm-level data. Table 4 presents the comparison between data and model-generated moments. The model successfully captures the mean gross leverage ratio, generating a value of 0.286 compared to 0.316 in the data. It also closely approximates the mean investment rate, producing 4.40 percentage points versus 3.94 in the data, and captures a substantial portion of the cross-sectional standard deviation of investment rates (8.27 in the model versus 10.26 in the data). The model also generates realistic persistence in firm financial structure, with a leverage autocorrelation of 0.989 that is close to the empirical value of 0.938. Additionally, the model produces a realistic fraction of firms with positive net debt (63.2% in the model versus 70.8% in the data).

The model's performance across these diverse moments, achieved without parameter tun-

Table 4: Model Fit

Moment	Description	Data	Model
$\mathbb{E}[\frac{b}{k}]$	Mean Gross Leverage	0.316	0.286
$\mathbb{E}[\widetilde{i}]$	Mean Investment Rate (p.p.)	3.936	4.398
$Corr(lev_t, lev_{t-1})$	Leverage Auto-correlation	0.938	0.989
$\sigma(\frac{i}{k})$	SD Investment Rate (p.p.)	10.263	8.27
Frac(b>0)	Share of Positive Net Debt	0.708	0.632
$\mathbb{E}[\mathrm{Exit}]$	Annual Exit Rate	0.08	0.08
$\frac{N_{age < 1yr}}{N_{age > 10yr}}$	Employment Size Ratio	0.022	0.2

ing to target specific statistics, provides confidence that the underlying economic mechanisms are well-captured. Moreover, the calibrated steady state shows firms on aggregate are net borrowers while households supply credits, which is qualitatively match the financial position discussion for a framework without government detailed in Appendix F. These external validation is particularly important for the credibility of the quantitative exercises that follow. It suggests the core mechanism (debt devaluation relaxing constraints) is correctly specified and that counterfactual predictions, such as how transitory inflation erodes real debt burdens differentially across leveraged firms, are credible, reflecting genuine economic forces embedded in the model's structure rather than artifacts of overfitting.

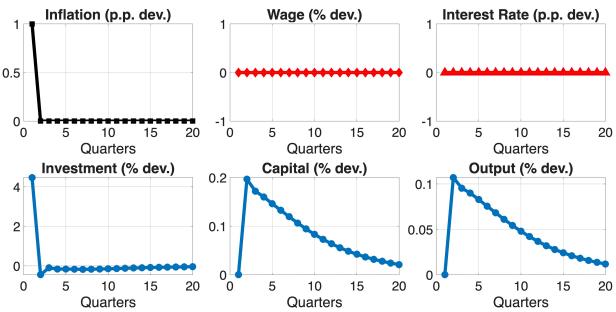
## 5.2 Quantitative Results

The calibrated model is initialized at its steady state and then hit by a transitory, unexpected 1% inflation shock.<sup>9</sup> Agents do not anticipate when setting nominal contracts in the last period, but immediately understand it is transitory and will not persist. I solve for the full transition path using the sequence-space Jacobian method, detailed in Appendix D. This method efficiently linearize around the steady state in sequence space, avoiding the curse of dimensionality for state space heterogeneous agent models with aggregate shocks. I report impulse response functions for both partial equilibrium and general equilibrium.

Figure 4 reports partial equilibrium responses holding wages and the real rate fixed so that the responses isolate the debt-inflation channel. It provides a clean benchmark for understanding the mechanical effect of debt revaluation before general equilibrium feedbacks. In the bottom-left panel, investment jumps on impact, rising approximately 4.5% above its steady-state level. The increase in the price level reduces the real burden of outstanding

<sup>&</sup>lt;sup>9</sup>In this section I do not model the structural source of the shock; the shock could reflect exchange-rate moves, changes in the central-bank inflation target, or commodity-price shocks.

Figure 4: Partial Equilibrium Impulse Response Function following 1% Unexpected Inflation



**Notes**: This figure plots partial equilibrium impulse responses to a one percent transitory unexpected inflation solved from calibrated model, with wages and real rates unchanged. Panels show percentage deviation from steady state; horizons in quarters.

nominal debt, effectively transferring resources toward more indebted and constrained firms, which triggers positive investment response. As the shock vanishes, the one-time debt relief is exhausted. Investment overshoots downward in the following periods before gradually returning to steady state. First, firms that invest heavily on impact now have larger capital stocks, reducing the optimal investment. Second, constraints re-tighten. Capital, being a predetermined stock, peaks with a one-period lag: it rises in the second period and is then gradually eroded as investment falls below replacement. Output co-moves with capital and labor inputs, rising in the short run as both increase. Taken in isolation, the inflation-only experiment produces sizable movements. In the data, however, such large deviations are typically attenuated by general equilibrium feedbacks. Most notably the endogenous responses of real rates should push against the partial equilibrium dynamics.

Figure 5 shows that the debt-inflation still produces a quantitatively meaningful short-run increase in investment in general equilibrium allowing for prices to adjust endogenously. The bottom-left panel shows that aggregate investment jumps by about 0.83% relative to steady state on impact. While this is far smaller, the effect remains economically meaningful. For comparison, a 25 basis point monetary policy shock would boost aggregate investment by 1.4% in Ottonello and Winberry (2020), suggesting the debt-inflation channel is quantitatively first-order. This aggregate response is especially notable when compared to

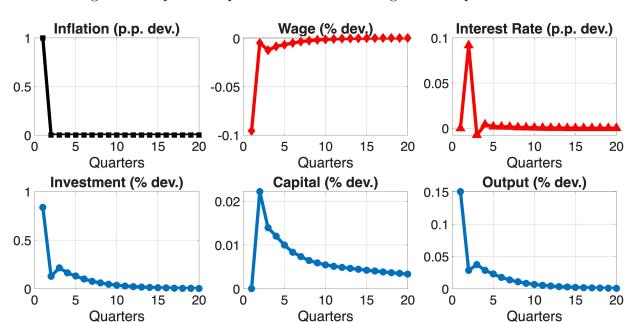


Figure 5: Impulse Response Function following 1% Unexpected Inflation

**Notes**: This figure plots general equilibrium impulse responses to a one percent transitory unexpected inflation solved from calibrated model. Panels show percentage deviation from steady state; horizons in quarters. Top middle panel shows real wages fall on impact due to consumption crowding-out, then gradually recover. Top right panel shows real interest rate rises endogeneously as investment demand increases, dampening the initial surge.

household-side estimates, which often found to be fairly modest Auclert (2019). The stark contrast highlights that firm-side effects could be essential for a complete understanding of the debt-inflation channel. Aggregate capital and output also rise, though more modestly than investment.

The top panels of Figure 5 display the core price dynamics that dampen the partial equilibrium investment surge. Inflation itself is transitory by construction, but the economy displays longer-lived adjustments in real wages and the real interest rate. A notable general-equilibrium outcome is that the real wage falls on impact despite higher investment-driven labor demand which would in turn push up wages in the top-middle panel. In my specification, higher investment crowds out contemporaneous consumption; with log utility this requires a decline in real wages to satisfy the household Euler condition. The top-right panel shows the real interest rate response (constructed according to the indexing convention described in Section 2). In the first period of shock, the ex post real rate should fall on impact, but realized variable does not change contemporaneous and future decisions. It is the immediate balance-sheet revaluation (not contemporaneous real rate effects) that drives

the investment jump; subsequently, the real interest rate rises endogenously as aggregate investment demand increases, raising the user cost of capital for all firms and crowding out investment for less-constrained firms, dampening and eventually reversing part of the initial surge.

These aggregate responses reflect the interaction of an initial redistribution and subsequent general-equilibrium feedbacks. The immediate investment impulse is driven mainly by policy-function shifts for constrained, indebted firms following debt devaluation. Persistence and any overshooting arise from distributional dynamics: as some constrained firms invest aggressively on impact, they accumulate capital and debt, moving toward the unconstrained region in subsequent periods. This endogenous 'graduation' from constraints naturally reduces the marginal propensity to invest and causes the investment boom to decay even without mean reversion in inflation. Others take on more debt to finance expansion, tightening their constraints in future periods. Prices adjust accordingly and push aggregates back to their steady state level.

The quantitative analysis demonstrates the debt-inflation channel generates economically significant macroeconomic effects. This finding has important implications for understanding inflation's real effects and for analysis during episodes of significant inflation surprises, such as the post-COVID period.

## 5.3 Anatomy of the Aggregate Response

The decomposition framework developed in equation 2 and 3 provides a way for understanding the sources of aggregate investment responses in the quantitative model. Using the calibrated steady-state distribution from Section 5.1, I compute the level effect (redistribution from household to firm), the heterogeneity effect (redistribution across firms), and the further decomposition into within- and between-firm-type components. This quantitative exercise reveals important insights about which margins drive aggregate dynamics.

Level Effect Dominates The quantitative model implies the average marginal propensity to invest  $\mathbb{E}[MPI] = 0.12$  and the average debt erosion  $\mathbb{E}\left[\frac{(1+R)b}{(1+\Pi)^2}\right] = 1.22$ , both strictly positive. The level effect capturing the redistribution from households to the corporate sector weighted by the average marginal propensity to invest is therefore positive and quantitatively large. Multiplying these terms yields a substantial first-order contribution to aggregate

 $<sup>^{10}</sup>$ In the model, firms are net debtors in aggregate while households are net savers, consistent with the institutional structure of corporate finance in the U.S. economy. This contrasts with some household models where agents can be on either side of the credit market. The positive average debt  $\mathbb{E}[b] > 0$  reflects that most firms use nominal debt financing, while the positive average MPI  $\mathbb{E}[MPI] > 0$  indicates that a meaningful fraction of firms face binding financial constraints in steady state.

investment from the pure cross-sectoral wealth transfer, independent of any heterogeneity across firms. In contrast, the unconditional covariance Cov(MPI,b) = -0.025, is small and slightly negative. This implies that the heterogeneity effect contributes negatively to the aggregate response, partially offsetting the mean effect. The aggregate investment increase following an unexpected inflation shock therefore comes almost entirely from the redistribution from households to firms, with the within-firm redistribution across heterogeneous firms playing a negligible or even slightly dampening role.

One may be concerned about that the negative covariance is counter intuitive and against the debt-inflation channel. To understand why the unconditional covariance is negative despite the positive within-firm-type relationship documented in the two-period model, I further employs equation 3 to decompose the unconditional covarince into to components. The within-firm-type component is  $\mathbb{E}_{z,k}[\text{Cov}(MPI, b \mid z, k)] = 0.106$ , confirming that holding productivity and capital constant, firms with higher debt have higher marginal propensities to invest due to tighter financial constraints. This captures the pure Fisher channel effect emphasized in the conceptual framework. However, the between-firm-type component is  $\operatorname{Cov}_{z,k}(\mathbb{E}[MPI \mid z,k],\mathbb{E}[b \mid z,k]) = -0.131$ , dominating the within-firm effect and driving the total covariance negative. This composition effect arises from systematic sorting in the steady-state distribution: high-productivity firms can borrow more, accumulate larger capital stocks, and maintain higher debt levels. But precisely because they are more productive and have higher net worth buffers, they are less likely to be financially constrained. However in the theory, the debt-inflation channel works through the constrained firms. Consequently, high-debt firms on average have lower marginal propensities to invest than low-debt firms, generating a negative cross-sectional correlation that offsets the positive conditional relationship. Appendix E provides more detailed discussion and plots by-productivity patterns of MPI and debt.

Implications for Aggregate Dynamics These findings have important implications for understanding how the debt-inflation channel operates in general equilibrium. The aggregate investment response to unexpected inflation is driven primarily by the mean redistribution from households to firms, not by the reallocation of resources across heterogeneous firms. The small negative unconditional covariance implies that firm heterogeneity, while generating rich cross-sectional patterns, does not amplify the aggregate effect. In fact, it slightly dampens it.

This does not mean firm heterogeneity is irrelevant. The positive within-firm-type component (conditional covariance) confirms that the Fisher channel operates as predicted: conditional on firm characteristics, higher debt leads to stronger investment responses. This

conditional relationship is similar to what the empirical strategy identifies by controlling the firm characteristics and exploiting within-firm variation over time. The negative composition effect (-0.131) arises from permanent differences across firm types in the steady state. The fact that the externally calibrated model correctly predicts this decomposition, including the sign reversal from conditional to unconditional covariances, provides strong validation of the underlying economic mechanism and underscores the importance of general equilibrium analysis for understanding aggregate implications.

## 5.4 Heterogeneous Responses

In order to better assess how well the calibrated model captures the firm-level patterns documented in Section 3, I simulate a panel of firms and estimate the same regression in equation 5 used in the empirical analysis. This validation tests whether the model's structural mechanisms can generate the cross-sectional heterogeneity. Simulation details are in Appendix D.

Several features of this exercise deserve emphasis. The model has only three state variables  $\{z, k, b\}$ , so it cannot mechanically generate every control variable used the empirical regression (e.g. GDP growth and federal funds rate). By contrast, firm-level idiosyncratic productivity paths are directly observable in the simulation and can be included as controls, giving the simulated regression somewhat more information than the empirical one. All specifications use firm and time two-way fixed effects only due to the one sector structure.

Table 5 summarizes the main comparison. Column (1) and (2) reproduces the empirical benchmarks from the Compustat sample. Columns (3) and (4) report estimates from the simulated panel under alternative specification choices. Qualitatively, the simulated regressions match the empirical signs: firms with higher indebtedness increase investment more following an unexpected inflation surprise. This confirms that the model's structural mechanism operates in the simulated data just as the theory predicts. Quantitatively, however, the simulated coefficients are smaller—roughly a factor of three to five smaller than the empirical estimates.

Several reasons may help to understand the quantitative differences. First, the simulated panel takes advantage of linear approximation which may miss higher-order effects. Second, measurement could matter: the empirical indebtedness proxy is constructed from accounting data while the model uses the structural state b without measurement error. Third, the simulated panel contains fewer omitted variables including those omitted variables in the empirical regression may help to amplify the reduced-form coefficient. Despite these quantitative differences, the key takeaway is that the model successfully reproduces the qualitative

Table 5: Empirical and Model Implied Heterogeneous Responses

Investment Rate	Empirical Estimate		Model Implied Results	
	(1)	(2)	$\overline{(3)}$	(4)
Unexp. Infl. $\times$ Indebt.	0.116***	0.124***	0.048*	0.024***
	(0.029)	(0.029)	(0.026)	(0.005)
Firm Control Observations $R^2$	No	Yes	No	Yes
	268757	268757	192801	192801
	0.118	0.125	0.272	0.968

**Notes:** Standard errors in parentheses. p < 0.10, p < 0.05, p < 0.01. Firm controls include interaction between real rate and financial position, productivity, size, sales growth and liquidity measures; all specifications include firm and time two way fixed effects. Results are estimated from regression specification 5. Simulated data ranges from 1995Q1 to 2023Q4 using real unexpected inflation series and model implied real rate series. Estimates of unexpected inflation or inflation series are absorbed by time fixed effects.

mechanism and generates coefficients of the right sign. Real-world factors not present in the model may strengthen the debt-inflation channel, implying a conservative model estimate of the channel's importance.

To study dynamic effects, I re-estimate the local-projection specification in equation 6 on the simulated panel and plot the cumulative responses in Figure 6. The simulated impulse-response profile matches the qualitative hump shape found in the data (Figure 3), but the magnitudes are substantially smaller. The 90% confidence bands are wide in the early horizons, and many point estimates are not statistically different from zero at conventional levels.

The simulation exercise demonstrates that the calibrated model successfully reproduces the key qualitative features of the empirical findings, which provides validation of the debtinflation channel and supports the aggregate quantification exercises. While the magnitudes are smaller in the model, this is not surprising given the parsimonious model structure and conservative calibration strategy.

# 5.5 Empirical Relevance: Two Episodes

To assess the empirical relevance of the firm-side debt-inflation, I feed the historical unexpected inflation series described in Section 3 into the model and compute the implied aggregate investment dynamics. Specifically, I use the sequence-space Jacobian method to construct investment responses as the linear combination of impulse responses with the

0.7 0.6 Interaction coefficient 0.5 0.4 0.3 0.2 0.1 0 0 2 3 5 6 7 8 1 9

Figure 6: Simulated Dynamic Impacts after the Inflation Surprise

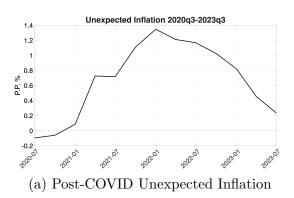
**Notes:** Plots the estimated coefficient  $\beta_h$  over h quarters following local projection regression (3), using model simulated data. Dashed lines report 90% error bonds.

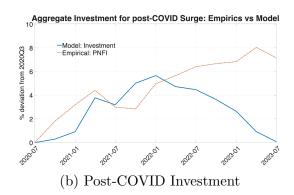
Quarters since shock

observed shock sequence. This isolates the contribution of debt revaluation to aggregate investment, abstracting from concurrent monetary policy reactions, supply-chain disruptions, fiscal interventions, and other contemporaneous shocks that affected the economy during these episodes. This approach answers a well-defined counterfactual question: how much of the observed investment dynamics can be explained by the debt-inflation channel alone? Two episodes, the post-COVID inflation surge and the Great Recession, are particularly interesting and informative.

The Post-COVID Investment Surge Figure 7 examines the post-COVID period, which featured a sequence of large positive inflation surprises unprecedented in recent decades. Panel (a) plots the unexpected inflation series from 2020Q3 to 2023Q3, showing a sustained period of positive surprises that peaked at approximately 1.4 percentage points in early 2022. The magnitude and persistence of these shocks stand out: after four decades of relatively stable inflation, unexpected inflation remained positive and large for nine consecutive quarters from 2021Q2 to 2023Q2. This prolonged sequence of positive surprises created cumulative debt relief for indebted firms far exceeding what would result from a single transitory shock. Panel (b) compares the model-implied investment path (blue line) with the cyclical component of real private non-residential fixed investment (PNFI) from FRED (orange dashed

Figure 7: The Post-COVID Inflation Surge





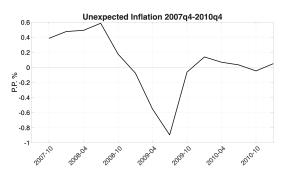
Notes: Panel (a) plots the unexpected inflation series (realized CPI inflation minus one-year-ahead expectations from Cleveland Fed) from 2020Q3 to 2023Q3. The series shows sustained positive surprises beginning in 2021Q2, peaking at approximately 1.4 percentage points in 2022Q1 as supply chain disruptions and fiscal stimulus drove prices above expectations, then gradually declining through 2023 as inflation moderated. Panel (b) compares model-implied aggregate investment (blue solid line) driven solely by the debt-inflation channel with the cyclical component of real private nonresidential fixed investment from FRED (orange dashed line), obtained using HP filter. Both series are normalized to 2020Q3 = 0. The model captures approximately 70% of the observed investment surge at its peak.

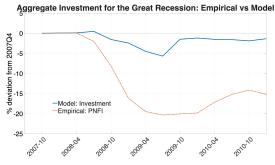
line), obtained using an HP filter<sup>11</sup>. Both series are normalized to 2020Q3 = 0 to facilitate comparison.

The model generates a substantial investment boom driven exclusively by the debt-inflation channel. The model-implied investment peaks at 5.66% above the 2020Q3 baseline in 2022Q1, capturing the qualitative upswing in investment activity during this period. The actual PNFI data shows a peak increase of 8.04% in 2023Q1, suggesting that the debt-inflation alone can account for up to 70% of the observed post-COVID investment surge at its peak. Comparing the trajectories, when unexpected inflation was increasing, the model tracks the data reasonably well with both series rising steadily. This explanatory power is economically significant and this finding contrasts sharply with conventional narratives that attribute the surge primarily to monetary and fiscal stimulus. While those factors undoubtedly played important roles, the debt-inflation channel appears to be quantitatively relevant. The model's ability to explain 70% investment surge using inflation surprises alone is notable, particularly when compared to the modest household-side Fisher effects documented in prior literature (Doepke and Schneider (2006), Auclert (2019)).

<sup>&</sup>lt;sup>11</sup>The use of the HP-filtered cyclical component removes low-frequency trends and focuses on business cycle fluctuations, making the comparison more appropriate for assessing the model's ability to explain short-to-medium run dynamics.

Figure 8: The Great Recession





- (a) The Great Recession Unexpected Inflation
- (b) The Great Recession Investment

Notes: Panel (a) shows the unexpected inflation series from 2007Q4 to 2010Q4. Beginning in 2008Q3, unexpected inflation turned sharply negative as the financial crisis triggered deflationary pressures, reaching a trough of approximately -0.9 percentage points in 2009Q1 when actual inflation fell well below expectations amid collapsing aggregate demand. The series then gradually recovered to near zero by 2010Q3. Panel (b) compares model-implied aggregate investment (blue solid line) driven by the debt-deflation channel with the cyclical component of real private nonresidential fixed investment from FRED (orange dashed line), both normalized to 2007Q4 = 0. The model accounts for approximately 25% of the observed investment collapse, with the remainder driven by credit market disruptions, elevated uncertainty, and aggregate demand shortfalls outside the model's scope.

The Great Recession Figure 8 examines the deflationary episode during the Great Recession, where the debt-inflation operates in reverse, which is debt-deflation in Fisher (1933). Falling prices increase the real debt burden, tightening financial constraints and depressing investment. Panel (a) shows the unexpected inflation series from 2007Q4 to 2010Q4. The series turns sharply in 2008Q4, reaching its trough of approximately -0.9 percentage points in 2009Q1. It then gradually recovers, returning to slightly positive territory by 2010Q3. This V-shaped pattern in deflation corresponds to the acute phase of the financial crisis and its immediate aftermath. Panel (b) presents the model-implied investment path alongside the cyclical component of real PNFI data, both normalized to 2007Q4 = 0. The model predicts a 5.68% decline in aggregate investment at the trough (2009Q2), driven mechanically by the deflationary debt-inflation elevating real debt burdens for constrained firms. The empirical PNFI data shows a far more severe collapse, declining by 20.36%. Consequently, the debt-deflation explains approximately 25% of the observed investment decline at the peak share.

Several factors may help to explain the divergent patterns between the model and data across the two episodes. For the post-COVID period, the model captures the initial surge well but predicts a reversal in late 2022-2023 that does not materialize in the data. First, the

model assumes one-period nominal debt that gets revalued immediately and then refinanced at market rates. In reality, many firms locked in low real interest rates by issuing longterm debt during 2020-2021, creating persistent balance sheet improvements that extended well beyond the inflation surprise period. A model with realistic multi-year debt maturity would generate more persistent effects. Second, the post-COVID period involved substantial real-side changes not modeled here, such as the relaxation of pandemic-related constraints, sector-specific demand shifts, and fiscal stimulus programs<sup>12</sup>. Third, the model abstracts from dynamic complementarities between debt relief and other investment determinants. Firms experiencing debt-inflation windfalls may have expanded capacity to hire workers and invest in intangibles, generating positive feedback loops not captured in the baseline framework. The model's linearization around steady state may also matter quantitatively. The sequence-space Jacobian method captures first-order effects but abstracts from nonlinearities. During the post-COVID boom, some severely constrained firms may have exhibited convex responses to large debt relief, investing disproportionately more as constraints relaxed substantially—effects the linear approximation misses. This could explain why the model underpredicts the peak by about 30%.

The model's explanatory power is weaker in this deflationary downturn than in the post-COVID inflationary expansion, revealing an important asymmetry. This asymmetry is natural and informative: in expansions driven by positive inflation surprises, the debt-inflation operates as a direct, first-order wealth transfer that immediately relaxes binding constraints, with few countervailing forces (beyond general equilibrium price adjustments). In severe recessions, by contrast, the debt-deflation is one of many contractionary forces, and it may be dominated by credit supply disruptions, uncertainty, aggregate demand shortfall and asset price drops that are outside the model's scope.

## 5.6 Qualitative Relevance

The debt-inflation emerges as a quantitatively robust mechanism for reallocating investment toward indebted firms following inflation surprises, with particularly strong effects during periods of sustained positive surprises like the post-COVID episode. The channel's explanatory power is more modest during severe downturns characterized by multiple concurrent shocks, yet even in the Great Recession, it accounts for roughly one-quarter of the investment collapse, which is still a non-trivial contribution.

These findings stand in sharp contrast to the household-side Fisher channel literature.

<sup>&</sup>lt;sup>12</sup>For example, the Bipartisan Infrastructure Law (2021) and CHIPS and Science Act (2022) directly stimulated investment in construction and semiconductor manufacturing.

Relative to the well-documented household-side Fisher effects, which Auclert (2019) finds to be modest and sometimes statistically indistinguishable from zero, these results point to a quantitatively meaningful firm-side debt-inflation for investment. The stark asymmetry of large positive investment effects versus negligible consumption effects, suggests that conventional wisdom about Fisher channels, built primarily on household evidence, has fundamentally overlooked where inflation's effects matter most. This calls for redirecting research focus from households to firms.

This asymmetry between household consumption and firm investment responses is likely to reflect differences in both the size of debt holdings (nonfinancial corporate debt is 72% of GDP versus household credit-to-GDP is about 68%<sup>13</sup>) and the sensitivity of expenditure decisions to net worth shocks: investment is likely to be more elastic due to financial constraints, while consumption is smoothed through precautionary savings.

The model's performance also suggests that the firm-side debt-inflation interacts importantly with monetary policy and other macroeconomic forces. During the post-COVID episode, accommodating monetary policy allows unexpected inflation consistently to hit the economy and boost investment. But monetary tightening from 2022Q2 likely dampened the Fisher effect through higher user costs and tighter expected future constraints, yet investment is still high. Even more aggressive monetary policy could further raise interest rate and mute debt-inflation effects with higher real rates. The debt-inflation channel in expansionary period can induce another subtle policy concern beyond the wage-price spiral. Unexpected inflation boosts investment through debt relief, which increases aggregate demand and can push inflation higher in subsequent periods, creating a self-reinforcing investment-inflation spiral. This amplification mechanism strengthens the case for aggressive disinflation during expansionary episodes, as allowing inflation surprises to persist may trigger accelerating dynamics through the firm debt-inflation channel.

During the Great Recession, by contrast, monetary policy was constrained by the zero lower bound, and credit supply disruptions likely amplified the debt-deflation beyond what the model predicts. Policymakers should actively intervene to generate positive inflation surprises, through forward guidance committing to overshooting inflation targets, aggressive quantitative easing, or fiscal-monetary coordination to activate the debt-inflation channel and accelerate recovery. Future research incorporating active monetary policy rules, occasionally binding zero lower bounds, and time-varying credit conditions could shed light on these policy interactions and refine the model's predictions during crisis episodes. Understanding how optimal policy should exploit or counteract the investment-inflation feedback loop in

<sup>&</sup>lt;sup>13</sup>See details in Appendix F and Federal Reserve

different states of the economy remains an important avenue for further work.

## 6 Conclusion

This paper combines micro-level evidence with a calibrated heterogeneous-firm general-equilibrium model to study how unexpected inflation affects corporate investment via the revaluation of nominal debt (the debt-inflation). Empirically, consistent with a simple conceptual framework, I document a robust finding from U.S. firm-level data that firms with different financial positions have differential investment responses to unexpected inflation. In particular, firms carrying larger nominal debt burdens tend to increase investment relatively more after a positive inflation surprise.

To identify mechanisms and quantify aggregate implications, I develop a tractable heterogeneous firm model with rigid nominal debt contracts and financing frictions. The model isolates the mechanism and quantifies its aggregate significance. Transitory inflation erodes the real value of outstanding nominal debt, relaxes financing constraints for a subset of indebted firms, and triggers an investment increase. In the baseline calibration, a 1% inflation surprise produces a non-negligible rise in aggregate investment by 0.83% and output 0.15%; simulated panels reproduce the cross-sectional heterogeneity observed in the data. When I map observed inflation surprises into the model, the debt-inflation explains a meaningful share (up to 70% peak-share) of the post-COVID investment surge, while it accounts for a smaller fraction of the Great Recession.

These findings elevate the firm-side debt-inflation as a quantitatively relevant mechanism for investment—complementary to the household-side Fisher effects found to be modest for consumption. They should, however, be read as upper-bound accounting for inflation-driven movements: policy tightening, price/wage adjustments, supply disruptions, and other shocks attenuate pass-through in the data. This study also opens several avenues for future research. The model simplifies the debt structure to one-period bonds; incorporating a richer term structure of corporate debt could reveal how maturity profiles mediate the strength and timing of the debt-inflation. Furthermore, embedding this mechanism in a framework with an endogenous monetary policy response would allow for a deeper analysis of optimal policy and the trade-offs between stabilizing prices and leveraging debt relief.

In conclusion, by shifting the focus from households to firms, the paper establishes the investment debt-inflation (Fisher) channel as a quantitatively important which is previously under-emphasized mechanism. Recognizing these firm-side effects is essential for a complete picture of how inflation shapes macroeconomic dynamics and for designing policies that

account for heterogeneity in corporate financial positions.

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# **Appendix**

# A Additional Empirical Results

This section of appendix provides the data description and various robustness checks.

#### A.1 Data Construction

This subsection describes the firm-level variables used in the empirical analysis in this paper.

#### Timing of Expected Inflation

The timing convention I employ addresses a subtle but important measurement issue. The Cleveland Fed's one-year-ahead expectation published in month t represents the expected inflation rate for the twelve months following that quarter's CPI release. This means that when month t+1's inflation is realized, the appropriate benchmark for forming an unexpected shock is the expectation that was published in t, after observing month t's inflation. This timing ensures that the expectation truly represents the "expected" component that firms would have incorporated into their planning, while the deviation captures the genuinely unexpected component of the inflation shock. This construction yields a cleaner identification of unexpected inflation compared to alternatives that might conflate predictable and unpredictable components.

#### Variables Descriptions.

Investment: defined as  $i_{j,t}$  of firm j at the end of period t. To get investment rate, I firstly calculate the series of real capital. For each firm, I set the first value of kj,t to be the level of gross plant, property, and equipment (ppegtq) in the first period in which this variable is reported in Compustat. From this period onwards, I compute the evolution of  $k_{j,t+1}$  using the changes of net plant, property, and equipment (ppentq), which is a measure net investment with significantly more observations than ppegtq (net of depreciation). If a firm has a missing observation of ppentq located between two periods with non-missing observations, I estimate its value using a linear interpolation with the values of ppentq right before and after the missing observation; if two or more consecutive observations are missing we do not do any imputation. We only consider investment spells with 20 quarters or more. Investment rate is calculated as the real investment expense divided by the last period capital stock.

Indebtedness: main proxy of indebtedness is defined as  $b_{j,t}$ , total debt (sum of dlcq and dlttq) and residualized to purge firm and time fixed effects. Alternative measures include

net indebtedness, leverage and net leverage. Net indebtedness is defined as the total debt minus cash and short-term investments (cash equivalents, cheq) in Compustat. Leverage is defined as the ratio of total debt to total assets (atq). Net leverage is defined as the ratio of total debt minus net current assets (actq minus lctq) to total assets.

Real sales growth: measured as log-differences in sales (saleq) deflated using the BLS implicit price deflator. Size: measured as the log of total real assets, deflated using the BLS implicit price deflator. Liquidity: defined as the ratio of cash and short-term investments (cheq) to total assets (atq). Cash flow: measured as EBIT divided by capital stock. Dividend payer: defined as a dummy variable taking a value of one in firm-quarter observations in which the firm paid dividends to preferred stock of the company (constructed using dvpq). Age: defined as the number of quarters appeared in the sample. Tobin's Q: defined as the market value of common stock (prccq\*cshoq) + book value of total assets (atq) – book value of common equity (ceqq), all divided by the book value total assets (atq).

Sectoral dummies: I consider the following sectors: (i) agriculture, forestry, and fishing: sic ; 999; (ii) mining: sic  $\in [1000; 1499];$  (iii) construction: sic  $\in [1500; 1799];$  (iv) manufacturing: sic  $\in [2000; 3999];$  (v) transportation, communications, electric, gas, and sanitary services: sic  $\in [4000; 4999];$  (vi) wholesale trade: sic  $\in [5000; 5199];$  (vii) retail trade sic  $\in [5200; 5999];$  (viii) services: sic  $\in [7000; 8999].$ 

Sample Selection Our empirical analysis excludes (in order of operation): 1. Firms in finance, insurance, and real estate sectors (sic  $\in$  [6000; 6799]), utilities (sic  $\in$  [4900; 4999]), nonoperating establishments (sic = 9995), and industrial conglomerates (sic = 9997). 2. Firms not incorporated in the United States. 3. Firm-quarter observations that satisfy one of the following conditions, aimed at excluding extreme observations: i. Negative capital or assets ii. Acquisitions (constructed based on aqcy) larger than 5% of assets. iii. Investment rate is in the top and bottom 0.5% of the distribution. iv. Investment spell is shorter than 40 quarters. v. Net current assets as a share of total assets higher than 10 or below -10. vi. Leverage higher than 10 or negative. vii. Quarterly real sales growth above 1 or below -1. viii. Negative sales or liquidity.

After applying these sample selection operations, I winsorize observations of leverage and distance to default at the top and bottom 0.5% of the distribution.

Table 6 summarizes the firm-level control variables, which are standard in the corporate finance literature. Notably, the average firm in our sample holds a low proportion of short-term debt (7.1%). This empirical fact suggests that the effects I document would likely be even stronger with a model featuring long-term debt, as the debt-inflation has more polr when debt maturity is longer. Our model simplifies this by assuming one-period debt, which

likely provides a conservative estimate of the channel's true strength.

Table 6: Firm Control Variables

Ctrl Variables	Mean	Median	S.D.	IQR
Sales Growth	0.011	0.014	0.222	0.167
Size	5.530	5.573	2.400	3.425
Curr Asset Ratio	0.516	0.520	0.247	0.392
Age (sample)	12.438	11.000	8.009	11.500
Profit to Capital	-0.238	0.039	6.430	0.147
ST Debt Ratio	0.071	0.013	0.260	0.054
Div Paying	0.091	0.000	0.288	0.000

### A.2 Sample Selection

I include two crisis periods in the main regression sample, which may in turn arouse suspicion that these periods can drive the main result. Table 7 presents the results with various samples. Column (3) excludes the two import recessions: the Great Recession and COVID recession and the point estimate does not change much in terms of magnitude and significance. The last column further excludes the whole post COVID period and the main estimate of interest declines but remain significant at 10% level. Overall, the main result is robust across various samples.

#### A.3 Interaction with Financial Variables

A key concern is whether the estimated interaction effect genuinely reflects debt devaluation or instead captures inflation's impact through other firm characteristics correlated with indebtedness. For instance, unexpected inflation might inflate nominal sales revenues, disproportionately benefiting firms with high operating leverage or market power. Alternatively, inflation could affect investment through cash flow channels, liquidity constraints, or life-cycle dynamics unrelated to balance sheets. To address these concerns, Table 8 augments the baseline specification with additional interaction terms between unexpected inflation and various firm-level controls: liquidity (cash holdings), firm size (log assets), age, and dividend payout ratios. I also explore alternative debt measures: net debt (gross debt minus cash) and long-term debt.

Across all specifications, the coefficient on the debt-inflation interaction term remains positive, statistically significant, and quantitatively stable—ranging from 0.065 to 0.127,

Table 7: Results for Various Samples

	(1)	(2)	(3)	(4)	(5)	(6)
Unexp Infl $\times$ Indebtedness	0.124***	0.108***	0.129***	0.127***	0.126***	0.076*
	(0.029)	(0.029)	(0.031)	(0.032)	(0.029)	(0.042)
Indebtedness	-0.414***	-0.423***	-0.354***	-0.413***	-0.372***	-0.269***
	(0.082)	(0.083)	(0.105)	(0.095)	(0.086)	(0.108)
Growth $\times$ Indebtedness	-0.045**	-0.037*	-0.057**	-0.043**	-0.055**	-0.074**
	(0.018)	(0.019)	(0.026)	(0.021)	(0.021)	(0.030)
$FFR \times Indebtedness$	-0.060***	-0.058**	-0.064***	-0.061***	-0.072***	-0.060**
	(0.020)	(0.023)	(0.020)	(0.020)	(0.020)	(0.021)
Observations	268757	244950	251150	255870	264037	232390
$R^2$	0.125	0.129	0.127	0.126	0.126	0.136
Firm Control	Yes	Yes	Yes	Yes	Yes	Yes
Time Sector FE	Yes	Yes	Yes	Yes	Yes	Yes
Time Clustering	Yes	Yes	Yes	Yes	Yes	Yes

**Notes:** Standard errors in parentheses. p < 0.10, p < 0.05, p < 0.01. Firm controls as indicated; all specifications include time-by-sector fixed effects; standard errors clustered by time. Results are estimated from regression specification 5. Column (1) is the main result. Column (2) considers only post 1994 sample. Column (3) excludes the Great Recession and COVID recession. (4) and (5) excludes the two recessions respectively. (6) only considers the pre-COVID sample and excludes the Great Recession.

compared to the baseline estimate of 0.124 in Table 2. This stability is economically meaningful: it indicates that the differential investment response between high- and low-debt firms persists even after controlling for inflation's potential effects through alternative channels. Column (1) is the main result without any more financial variable interactions with inflation surprises. Columns (2)- (5) control for interactions with liquidity, size, age, and dividend policy while retaining the baseline debt measure. The debt interaction coefficient remains highly significant in all cases (0.082-0.119), demonstrating that the debt-inflation operates independently of these firm characteristics. Notably, Liquidity result shows the opposite sign and operates in an opposite way of the debt-inflation channel. This is intuitive because if the firm holds more cash or equivalents, it will lose real value with positive inflation surprises. The fact that controlling for size (Column 3, capturing scale effects) and age (Column 4, capturing life-cycle dynamics) does not attenuate the debt coefficient suggests the Fisher effect is not merely proxying for larger or more mature firms having different inflation sensitivities. Similarly, controlling for dividend payout (Column 6) confirms the result is not driven by payout policy differences across high- and low-debt firms. Together, these results establish that the debt-inflation interaction captures a distinct mechanism of the revaluation of nominal debt, rather than reflecting omitted firm attributes or alternative transmission channels.

Table 8: Results Controlling for Interaction with Other Financials

	Debt	Liquidity	Size	Age	Div
Unexp Infl $\times$ Indebtedness	0.124**	0.082***	0.109***	0.109***	0.119***
Unexp Infl $\times$ Liquidity	(0.029)	(0.030) $-0.127***$ $(0.036)$	(0.032)	(0.027)	(0.029)
Unexp Infl $\times$ Size		,	0.020		
			(0.035)	0.000**	
Unexp Infl $\times$ Age				0.003** $(0.001)$	
Unexp Infl $\times$ Div Pay				(0.001)	0.239*** (0.069)
Observations	268,757	254,991	255,045	255,045	255,045
$R^2$	0.125	0.128	0.126	0.129	0.126

**Notes:** Standard errors in parentheses. \* p < 0.10, \*\* p < 0.05, \*\*\* p < 0.01. Two-way clustering by firm and time. Firm, sector-time FE and aggregate controls included. All controls interact with GDP growth and the federal funds rate.

#### A.4 Alternative Measures

Table 9: Results Controlling for Interaction with Debt-Related Financials

	Debt	NetDebt	LongDebt
Unexp Infl × Indebtedness	0.124**	0.065**	0.077***
	(0.029)	(0.028)	(0.026)
Indebtedness	-0.414***	-0.492***	-0.229***
	(0.082)	(0.078)	(0.075)
Growth $\times$ Indebtedness	-0.045**	-0.017*	-0.049***
	(0.018)	(0.019)	(0.018)
$FFR \times Indebtedness$	-0.060***	-0.020	-0.041
	(0.020)	(0.015)	(0.018)
Observations	268757	179450	236406
$R^2$	0.125	0.140	0.130

**Notes:** Standard errors in parentheses. \* p < 0.10, \*\* p < 0.05, \*\*\* p < 0.01. Two-way clustering by firm and time. Firm, sector-time FE and aggregate controls included. Net debt and long debt columns replace the debt in the main specification. All controls the interaction with GDP growth and federal funds rate.

In the main context, I use logarithm of debt as the main proxy for the indebtedness. In this subsection, I also show robust results using other debt variables. Table 9 presents results with net debt and long-term debt as the proxy for firm's indebted position. Both variables are constructed following the same logic as the debt in the main context. Column (1) is the main result. Column (2) and (3) show the corresponding results with net debt and long term debt. Using net debt yields a coefficient of 0.065, slightly smaller than the baseline, likely because netting out liquid assets partially offsets the debt burden that inflation erodes. Long-term debt is also informative, generating a coefficient of 0.077. Long-term debt has fixed nominal repayment schedules extending multiple years into the future, making it most exposed to inflation surprises. Though the two coefficients are not directly comparable, the finding that long-term debt aligns precisely with the model's emphasis on predetermined nominal contracts as the source of the Fisher effect.

Due to the mechanism of debt-inflation, the level variable is naturally more advantageous than the ratio. But in the literature, leverage ratio is a commonly used proxy for financial position, therefore I also provide the sample statistics and regression results using leverage ratio. Table 10 shows the statistics of the sample with leverage ratio. Table 11 and figure 9 present the corresponding regression result and dynamic plot. On impact estimates are significant and precise but the following dynamic effects do not show similar persistency like

in figure 3

Table 10: Firm Investment Summary

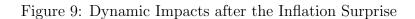
Statistic	$\Delta \log k_{jt+1}$	$i_t$	$\Delta \log(ppe)_{jt+1}$	$capx_t$	$lev_{t-1}$
Mean	0.490	4.107	0.490	9.088	0.269
Median	-0.441	2.743	-0.466	4.343	0.201
S.D.	8.983	10.643	14.480	542.663	0.371
95th percentile	12.031	15.986	16.568	22.453	0.751
Observations	316147	316147	315334	313402	316147

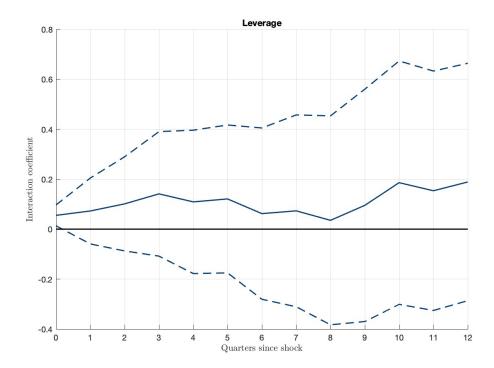
Notes: Because taking logarithm will naturally omit zero debt firms, the regression sample using leverage ratio is significantly larger than that using log debt. Other than observations, other sample statistics are similar to table 1

Table 11: Results using Leverage

Investment Rate	(1)	(2)	(3)	(4)
Unexpected inflation $\times$ Lev	0.055*	0.056*		
	(0.030)	(0.030)		
Inflation $\times$ Lev			$0.052^{**}$	$0.054^{**}$
			(0.024)	(0.024)
Leverage	-0.484***	-0.357***	-0.562***	-0.439***
	(0.074)	(0.072)	(0.086)	(0.086)
GDP Growth $\times$ Lev	-0.045***	-0.045***	-0.047***	-0.047***
	(0.015)	(0.015)	(0.016)	(0.015)
$FFR \times Lev$	-0.045**	-0.036**	-0.058***	-0.050***
	(0.018)	(0.017)	(0.019)	(0.018)
Observations	316147	316147	316147	316147
$R^2$	0.110	0.117	0.110	0.117
Firm Control	No	Yes	No	Yes
Time Sector FE	Yes	Yes	Yes	Yes
Two-way Clustering	Yes	Yes	Yes	Yes

Notes: Standard errors in parentheses. \*p < 0.10, \*\*p < 0.05, \*\*\*p < 0.01. Firm controls as indicated; all specifications include time-by-sector fixed effects; standard errors clustered by time and firm. Results are estimated from regression specification 5. First two columns employ unexpected inflation series  $\varepsilon_t^{\pi}$  constructed above, while third and fourth column show results directly using CPI inflation.  $\mathbf{Z_{j,t}}$  is a vector of control variables at firm level and interaction of demeaned financial position with GDP growth and federal funds rate. Estimates of unexpected inflation or inflation series are absorbed by time fixed effects. This table show robust results in addition to table 2





**Notes:** Plots the estimated coefficient  $\beta_h$  over h quarters following local projection regression (3) using leverage ratio as the proxy of indebtedness. Dashed lines report 90% error bonds. Estimation of cumulative capital change is different from the investment rate from on impact regression. We can regard the slop of the graph as the investment responses and in the 8th quarter, the positive investment responses vanish.

# B Two-Period Model Capital Structure

**Setup.** Set  $\delta = 1$  and  $\Pi_2 = 1$ . Period-1 net worth after repaying old nominal debt is

$$nw_1 = k_1^{\alpha} - \frac{1+r}{\Pi_1} b_1 \tag{21}$$

The nonnegative dividend constraint implies

$$d_1 \ge 0 \quad \Longleftrightarrow \quad k_2 - b_2 \le n w_1 \tag{22}$$

The borrowing (collateral) constraint is

$$(1+r)b_2 \le \phi k_2^{\alpha} \iff b_2 \le \frac{\phi}{1+r} k_2^{\alpha} \tag{23}$$

Combining (22)–(23) yields the single feasibility inequality

$$h(k_2) \equiv k_2 - \frac{\phi}{1+r} k_2^{\alpha} \le nw_1$$
 (24)

Unconstrained benchmark. With  $\delta = 1$ , the unconstrained optimum  $k_2^u$  satisfies

$$1 = \frac{1}{1+r} \alpha (k_2^u)^{\alpha - 1} \implies k_2^u = (\frac{\alpha}{1+r})^{\frac{1}{1-\alpha}}$$
 (25)

Evaluate the feasibility function at  $k_2^u$ :

$$h(k_2^u) = k_2^u - \frac{\phi}{1+r} (k_2^u)^\alpha = k_2^u \left[ 1 - \frac{\phi}{\alpha} \right], \text{ since } \frac{1}{1+r} (k_2^u)^\alpha = \frac{k_2^u}{\alpha}.$$
 (26)

**Implication.** The sign of  $h(k_2^u)$  depends only on  $\alpha - \phi$ :

- If  $\alpha > \phi$ , then  $h(k_2^u) > 0$ . To implement  $k_2^u$  the firm needs strictly positive net worth  $nw_1 \geq h(k_2^u) > 0$ . Firms with lower (but feasible)  $nw_1 \in [0, h(k_2^u))$  cannot reach  $k_2^u$ ; their optimum is constrained on the boundary  $k_2 \frac{\phi}{1+r}k_2^{\alpha} = nw_1$ . This creates a nonempty constrained region and a non-neutral Fisher effect on impact.
- If  $\alpha \leq \phi$ , then  $h(k_2^u) \leq 0$ . Because feasible firms satisfy  $nw_1 \geq 0$ , they automatically meet  $nw_1 \geq h(k_2^u)$  and choose  $k_2^u$  (unconstrained); the constrained region vanishes on impact.

Choosing  $\phi < \alpha$  ensures  $h(k_2^u) > 0$  by (A6), so for realistic  $nw_1$  some firms are constrained while others are not. In the two-period illustrations, I set  $\phi = 0.6$  and (e.g.)  $\alpha = 0.8$ , which

guarantees: (i) a nonempty constrained set  $nw_1 \in [0, h(k_2^u))$ ; (ii) heterogeneous responses to inflation across indebtedness  $b_1$ ; and (iii) a strictly positive on-impact slope  $\partial k_2^*/\partial \pi_1$ . This choice delivers the non-neutrality.

# C Firm Optimality Conditions in Quantitative Model

This appendix provides a complete derivation of the firm's first-order conditions (FOC). I also present envelope relations, the full Kuhn-Tucker characterization, and a local comparative-static result that signs the effect of a small, unexpected inflation surprise on the borrowing multiplier  $\psi_t$ , the dividend multiplier  $\xi_t$ , and contemporaneous investment  $i_t$ .

## C.1 Lagrangian

A continuing firm (index suppressed) solves

$$V_t^c(z, k, b) = \max_{k', b'} \left\{ d_t + \mathbb{E}_t[\Lambda_{t, t+1} V_{t+1}(z', k', b')] \right\},\,$$

subject to the dividend identity (real terms)

$$d_t = p_t z_t k_t^{\alpha} n_t^{\nu} - w_t n_t - i_t - AC(i_t, k_t) - (1 + R_t) \frac{b_t}{\Pi_t} + b_{t+1},$$
(27)

with  $i_t = k' - (1 - \delta)k$  and  $AC(i, k) = \frac{\gamma}{2} \frac{i^2}{k}$ .

The borrowing (collateral) limit is

$$b_{t+1} \le \mathcal{B}_t(k_{t+1}) \equiv \phi \cdot \frac{\prod_{t+1}}{1 + R_{t+1}} \Big( p_{t+1} \, \underline{z}_{t+1} \, k_{t+1}^{\alpha} n_{t+1}^{\nu} - w_{t+1} n_{t+1} + (1 - \delta) k_{t+1} \Big), \tag{28}$$

where  $\phi > 0$  measures credit-market tightness (higher  $\phi$  increases allowable borrowing). I set  $\phi = 1$  as a normalization; here keep it explicitly.

Introduce multipliers:  $\xi_t \geq 0$  on the dividend non-negativity constraint  $d_t \geq 0$ , and  $\psi_t \geq 0$  on the borrowing constraint  $\mathcal{B}_t(k') - b' \geq 0$ . The continuation Lagrangian is

$$\mathcal{L}_{t} = d_{t} + \mathbb{E}_{t}[\Lambda_{t,t+1}V_{t+1}(z',k',b')] + \xi_{t} d_{t} + \psi_{t}(\mathcal{B}_{t}(k') - b').$$

#### C.2 First-order conditions

Differentiate  $\mathcal{L}_t$  w.r.t. choice variables b' and k'. Expectations are conditional on information at t.

FOC w.r.t. 
$$b'$$

$$\frac{\partial \mathcal{L}_t}{\partial b'}: \quad 1 + \mathbb{E}_t[\Lambda_{t,t+1}V_{b'}(z',k',b')] - \psi_t = 0.$$

Using envelope (below)  $V_{b'}(z', k', b') = -\frac{1 + R_{t+1}}{\Pi_{t+1}} (1 + \xi_{t+1})$ , we obtain

$$1 = \mathbb{E}_t \left[ \Lambda_{t,t+1} \frac{1 + R_{t+1}}{\Pi_{t+1}} (1 + \xi_{t+1}) \right] + \psi_t. \tag{29}$$

Complementarity:

$$\psi_t \ge 0, \qquad \mathcal{B}_t(k') - b' \ge 0, \qquad \psi_t(\mathcal{B}_t(k') - b') = 0.$$

**FOC w.r.t.** k'. Because  $i_t = k' - (1 - \delta)k$  and  $\partial AC/\partial k' = \partial AC/\partial i \cdot \partial i/\partial k' = \gamma i_t/k_t$ , differentiation yields

$$(1 + \xi_t) \frac{\partial d_t}{\partial k'} + \mathbb{E}_t [\Lambda_{t,t+1} V_{k'}(z', k', b')] + \psi_t \frac{\partial \mathcal{B}_t(k')}{\partial k'} = 0.$$

Since  $\partial d_t/\partial k' = -1 - \gamma \frac{i_t}{k_t}$ , rearrange to

$$(1+\xi_t)\left(1+\gamma\frac{i_t}{k_t}\right) = \mathbb{E}_t[\Lambda_{t,t+1}V_{k'}(z',k',b')] + \psi_t \frac{\partial \mathcal{B}_t(k')}{\partial k'}.$$
 (30)

Remarks: - The multiplicative factor  $1+\xi_t$  multiplies the marginal current cost of investing; if  $\xi_t = 0$  we recover the simplified condition often used in text. - The term  $\psi_t \partial_{k'} \mathcal{B}_t$  captures how increasing k' relaxes future borrowing capacity (via collateral) and thus raises the marginal value of investment when  $\psi_t > 0$ .

#### Envelope conditions

By the envelope theorem,

$$V_b(z, k, b) = \frac{\partial d_t}{\partial b} (1 + \xi_t) = -\frac{1 + R_t}{\Pi_t} (1 + \xi_t).$$
 (31)

And formally<sup>14</sup>,

$$V_{k}(z,k,b) = (1+\xi_{t}) \left( p_{t} z \alpha k^{\alpha-1} n^{\nu} - \partial_{k} A C(i,k) \right)$$

$$+ \mathbb{E}_{t} \left[ \Lambda_{t,t+1} \left( V_{k'}(z',k',b') \partial_{k} g_{k}(z,k,b) + V_{b'}(z',k',b') \partial_{k} g_{b}(z,k,b) \right) \right]$$

$$+ \mathbb{E}_{t} \left[ \Lambda_{t,t+1} \left( \frac{\partial V_{t+1}}{\partial p_{t+1}} \partial_{k} p_{t+1} + \frac{\partial V_{t+1}}{\partial w_{t+1}} \partial_{k} w_{t+1} + \frac{\partial V_{t+1}}{\partial R_{t+1}} \partial_{k} R_{t+1} + \cdots \right) \right],$$
(32)

The explicit expansion of  $V_{k'}$ 

<sup>&</sup>lt;sup>14</sup>real relative price p does not change in my flexible price model, but with nominal rigidities, it will.

#### Kuhn-Tucker conditions

$$\xi_t \ge 0, \quad d_t \ge 0, \quad \xi_t d_t = 0,$$

$$\psi_t \ge 0$$
,  $\mathcal{B}_t(k') - b' \ge 0$ ,  $\psi_t(\mathcal{B}_t(k') - b') = 0$ .

## C.3 Sign of $\partial \psi / \partial \Pi$ , $\partial \xi / \partial \Pi$ , and $\partial i / \partial \Pi$

I now linearize the key FOC+KKT system to sign the responses of  $\psi_t$ ,  $\xi_t$ , and  $i_t$  to a small, unexpected increase in gross inflation  $\Pi_t$ . Keep  $\phi$  explicit: since  $\mathcal{B}_t$  is proportional to  $\phi$  (see (28)),  $\phi$  scales  $\partial_{k'}\mathcal{B}_t$  and thus the sensitivity of investment to  $\psi_t$ .

**Key identities for linearization.** Write the core (nonlinear) system as

$$F_1(\psi_t, \xi_{t+1}, \Pi_{t+1}, R_{t+1}, \dots) \equiv 1 - \mathbb{E}_t \left[ \Lambda_{t,t+1} \frac{1 + R_{t+1}}{\Pi_{t+1}} (1 + \xi_{t+1}) \right] - \psi_t = 0, \tag{33}$$

$$F_2(\psi_t, \xi_t, k', b', i_t, \dots) \equiv (1 + \xi_t)(1 + \gamma i_t/k_t) - \mathbb{E}_t[\Lambda_{t,t+1}V_{k'}] - \psi_t \partial_{k'} \mathcal{B}_t(k') = 0, \quad (34)$$

$$F_3(d_t, \xi_t) \equiv d_t = 0$$
 (for firms with binding dividend constraint). (35)

Here  $F_3$  expresses the binding dividend condition  $d_t = 0$  for firms with  $\xi_t > 0$  pre-shock. For unconstrained firms use  $d_t > 0$ ,  $\xi_t = 0$ .

Total differentiation. For a constrained-firm (pre-shock  $\xi_t > 0$ ,  $\psi_t \ge 0$ ) take total differentials of (33)–(35) around the pre-shock steady state and collect unknowns

$$x \equiv (d\psi_t, d\xi_t, d\xi_{t+1}, dk_{t+1}, dk_{t+1}, di_t, \ldots)^{\top}.$$

We can write the linear system as

$$Ax = B \, \mathrm{d}\Pi_t, \tag{36}$$

where A is the Jacobian matrix of partial derivatives of  $F \equiv (F_1, F_2, F_3, ...)$  w.r.t. the unknown vector x, and B collects direct partials of F with respect to  $\Pi_t$ . In particular the entry of B corresponding to  $F_3$  (the dividend eq.) contains

$$\frac{\partial d_t}{\partial \Pi_t} = (1 + R_t) \frac{b_t}{\Pi_t^2} > 0 \quad \text{(for } b_t > 0\text{)}.$$

Sufficient sign conditions. Under standard assumptions—(i) the Jacobian A is nonsingular (so the implicit function theorem applies), (ii) the Jacobian has economically plausible

sign pattern we obtain

$$x = A^{-1}B \,\mathrm{d}\Pi_t$$

so the sign of each component of x is given by the sign of the corresponding element of  $A^{-1}B$ .

Intuition and sign results. The direct effect of  $d\Pi_t > 0$  is to raise contemporaneous real dividends (term  $\partial d/\partial \Pi_t > 0$ ); to keep  $d_t = 0$  for a constrained firm, other variables must adjust. Two adjustment channels dominate:

- 1. **Debt Euler** / **expectation channel:** a larger  $\Pi_t$  reduces the real burden of nominal debt  $(1+R_t)b_t/\Pi_t$  and also reduces the expected real repayment factor  $(1+R_{t+1})/\Pi_{t+1}$ , lowering  $\mathcal{E}_t$  in equation (33). Since  $1 = \mathcal{E}_t + \psi_t$ , a fall in  $\mathcal{E}_t$  implies  $\psi_t$  must fall:  $d\psi_t < 0$ .
- 2. Dividend multiplier channel: increased real dividends directly relax the binding dividend constraint, implying  $\xi_t$  must fall to restore the constraint. Thus  $d\xi_t < 0$ .

Given  $d\psi_t < 0$  and  $d\xi_t < 0$ , the RHS of the investment FOC (34) declines (because the term  $\psi_t \partial_{k'} \mathcal{B}_t$  becomes smaller and  $(1 + \xi_t)$  decreases), implying the required discounted marginal payoff from investing falls; therefore the optimal  $i_t$  increases:  $di_t > 0$ .

Assume the Jacobian A defined by the FOC+KKT system is nonsingular and that  $\partial_{k'}\mathcal{B}_t(k') > 0$ . For firms with binding dividend constraint  $(d_t = 0, \xi_t > 0)$  and binding or near-binding borrowing constraint, a small unexpected positive change in gross inflation  $\Pi_t$  leads to

$$\frac{\partial \psi_t}{\partial \Pi_t} < 0, \qquad \frac{\partial \xi_t}{\partial \Pi_t} < 0, \qquad \frac{\partial i_t}{\partial \Pi_t} > 0.$$

## D Solution Method

### D.1 Steady-State Equilibrium Solution

In this section, I outline the numerical algorithm used to solve for the steady-state equilibrium of the model. I solve for the stationary firm distribution and the corresponding equilibrium prices and policy functions using value function iteration on a discretized state space.

The state space for an individual firm is given by its idiosyncratic productivity z, capital stock k, and real net debt position b. I discretize the state space as follows:

- Productivity (z): The AR(1) process for idiosyncratic productivity is discretized into  $N_z = 7$  states using the Tauchen (1986) method, as adapted by Terry and Knotek II (2011).
- Capital (k) and Debt (b): I specify discrete grids for capital (120) and debt (100). The grids are designed to be sufficiently wide to ensure the bounds are non-binding and feature more points concentrated in regions where the value function exhibits more curvature, such as near the borrowing and dividend constraints.

The computational algorithm proceeds as follows:

- 1. **Initialization:** Guess a set of steady-state aggregate prices, specifically the real wage w and the real interest rate r. The household's stochastic discount factor is then given by  $\Lambda = \beta$ .
- 2. Value Function Iteration (Inner Loop): Given the steady-state real interest rate  $\{r\}$ , which is pinned down by the household's discount factor, guess a real wage  $\{w\}$ , solve the firm's dynamic programming problem to find the optimal policy functions for investment k'(z, k, b) and borrowing b'(z, k, b), and the associated value function V(z, k, b).
  - (a) Start with an initial guess for the value function,  $V_0(z, k, b)$ .
  - (b) Iterate on the Bellman equation until convergence:

$$V_{j+1}(z, k, b) = \max_{k', b'} \{ d(z, k, b, k', b') + \beta \mathbb{E}[V_j(z', k', b')|z] \}$$

subject to the non-negative dividend and borrowing constraints. This step yields the converged value function  $V^*(z, k, b)$  and the policy functions  $k^*(z, k, b)$  and

 $b^*(z, k, b)$  for the given prices. The iteration stops when the sup-norm distance between successive value functions,  $||V_{j+1} - V_j||_{\infty}$ , is smaller than a specified tolerance level  $\epsilon_V$ .

3. **Distribution Calculation:** With the converged policy functions, compute the stationary (ergodic) distribution of firms,  $\mu(z, k, b)$ , across the state space. This is done by iterating on the law of motion for the distribution using Young (2010) until it converges to a fixed point.

#### 4. Market Clearing (Outer Loop):

- (a) Using the stationary distribution  $\mu(z, k, b)$  and the policy functions, compute the aggregate quantities for capital (K), labor (N), investment (I), and net debt (B).
- (b) Check if the labor markets clear at the guessed prices  $\{w\}$ . Specifically, verify if the intra-temporal Euler equation holds.
- (c) If not, update the guessed prices  $\{w\}$  in the direction of the excess demands and return to step 2.
- 5. Convergence: The algorithm has converged when the market-clearing conditions are satisfied within a specified tolerance  $\epsilon_M$ . The final output includes the steady-state equilibrium prices  $\{w^*, r^*\}$ , aggregate quantities  $\{C^*, I^*, K^*, N^*\}$ , the firm value function  $V^*(z, k, b)$ , policy functions  $\{k^*(z, k, b), b^*(z, k, b), n^*(z, k, b)\}$ , and the stationary distribution  $\mu^*(z, k, b)$ .

# D.2 Transitional Dynamics Solution

To analyze the economy's response to an unexpected inflation shock, I compute the model's transitional dynamics using the sequence-space Jacobian method developed by Auclert et al. (2021). This method is highly efficient for solving heterogeneous-agent models with aggregate shocks, as it avoids the need to repeatedly simulate the full distribution of firms over time. The approach linearizes the model's equilibrium conditions around the steady state and solves for the entire time path of aggregate variables and firm-level decisions simultaneously.

The computational procedure is as follows:

1. **Define the Sequence-Space Equilibrium:** First, the dynamic model is cast in sequence space. The unknowns are the infinite sequences of aggregate variables, prices, and the distribution of firms,  $\{Y_t, C_t, I_t, K_t, w_t, r_t, \mu_t\}_{t=0}^{\infty}$ , in response to a sequence of

aggregate shocks. The equilibrium is characterized by a system of equations representing market clearing, households' and firms' optimality conditions, and the law of motion for the firm distribution for all periods  $t \geq 0$ .

- 2. Linearize the System: The system of equations is linearized around the deterministic steady state. This involves computing the derivatives of the equilibrium conditions with respect to the sequence of endogenous and exogenous variables. A key advantage of the sequence-space Jacobian method is that it provides techniques to efficiently compute these derivatives (Jacobians) without iterating on the value function. The derivatives of the firm-level policy rules with respect to aggregate variables are crucial inputs.
- 3. Construct the Sequence-Space Jacobian: The core of the method is the construction of the Jacobian matrix,  $\mathcal{J}_x$  and  $\mathcal{J}_z$ , which represents the stacked, linearized equilibrium conditions.

When computing the Jacobians, I firstly solve the backward iteration given the specific price shock. Using two-sided numerical differentiation technique, I can effectively calculate the policy changes and the resulting  $\mathcal{Y}$  in this process Then I forward iterate to get the distributional change.

Secondly, I follow Auclert et al. (2021) to compute the expectation vectors and construct Fake News Matrix  $\mathcal{F}$  by summarizing  $\mathcal{Y}$  and the inner product of expectation vectors and distributional changes.

Finally, I build up the Jacobian matrix with each entity  $\mathcal{J}_{t,s} = \mathcal{J}_{t-1,s-1} + \mathcal{F}_{t,1}$ .

4. Solve for the Transition Path: This economy can be effectively summarized by three equations:

$$\mathbf{H}(\mathbf{U}, \mathbf{Z}) = \begin{pmatrix} \frac{\frac{1}{\mathbf{C_t}} - \beta(\mathbf{1} + \mathbf{r_{t+1}}) \frac{1}{\mathbf{C_{t+1}}} \\ \mathbf{w_t} - \chi \mathbf{C_t} \\ \mathbf{Y_t} - \mathbf{C_t} - \mathbf{I_t} - \mathbf{A}\mathbf{C_t} - \bar{\mu}_{\mathbf{ent}} \mathbf{k_0} + \pi_{\mathbf{d}} (\mathbf{1} - \delta) \mathbf{K_t} \end{pmatrix} = 0$$

Where:

$$\mathbf{U} = (\mathbf{w}, \mathbf{r}, \mathbf{C}) \qquad \mathbf{Z} = \mathbf{\Pi}$$

and  $X = \{Y, I, AC, K\}$  are all functions M(U, Z) from firm block.

The system is solved by inverting the Jacobian matrix to find the economy's impulse

response functions (IRFs):

$$d\mathbf{U} = -\mathbf{H}_u^{-1}\mathbf{H}_z \cdot d\mathbf{Z}$$
 
$$d\mathbf{X} = \mathbf{M}_u d\mathbf{U} + \mathbf{M}_z d\mathbf{Z} = \mathbf{G} d\mathbf{Z}$$

where  $d\mathbf{U}$  is the vector of changes of endogenous variable (w, r, C) from their steadystate values over time.  $d\mathbf{X}$  is the vector of percentage deviations of all other aggregate variables from their steady-state values, and  $d\mathbf{Z}$  is the vector representing the sequence of exogenous shocks (in this case, the one-time, unexpected 1% inflation shock at t = 0). This single matrix inversion yields the complete, linear approximation of the transition path for all aggregate variables (Investment, Output, Capital, etc.) following the shock.

This approach cleanly isolates the effects of the debt-inflation. By feeding a one-time, transitory inflation surprise into the model, I can trace out its real effects on the economy while holding all other potential shocks constant. The resulting impulse response functions, as shown in Figures 4 and 5, illustrate the dynamic adjustment of the economy as the initial wealth redistribution from the debt-inflation propagates through general equilibrium feedbacks.

# E Aggregation in Steady State

In this appendix, I provide the technical details supporting the heterogeneity analysis in Section 5.3.

## E.1 Computing the Marginal Propensity to Invest (MPI)

The marginal propensity to invest (MPI) measures the sensitivity of a firm's investment to a change in its net worth. Formally, we define it for firm i as:

$$MPI_i = \frac{\partial k_i'}{\partial nw_i}$$

where  $k'_i$  is the firm's choice of capital for the next period and  $nw_i$  is its current-period net worth.

Since the model uses debt  $b_i$  as a state variable instead of net worth, I compute the MPI using the chain rule:

$$MPI_i = \frac{\partial k_i'}{\partial nw_i} = \frac{\partial k_i'/\partial b_i}{\partial nw_i/\partial b_i}$$

From the definition of net worth,  $nw_i = (1 - \delta)k_i + F(z_i, k_i) - \frac{1+R}{1+\pi}b_i$ , it follows that debt and net worth are inversely related:  $\frac{\partial nw_i}{\partial b_i} = -\frac{1+R}{1+\pi}$ . Substituting this into the chain rule yields the formula for MPI:

$$MPI_i = -\frac{1+\pi}{1+R} \cdot \frac{\partial k_i'}{\partial b_i}$$

In the steady state where inflation  $\pi = 0$ , this simplifies to:

$$MPI_i = -\frac{1}{1+R} \cdot \frac{\partial k_i'}{\partial b_i}$$

I compute the derivative  $\partial k'_i/\partial b_i$  numerically. For interior points  $b_j$  on the debt grid, I use a central difference approximation:

$$\left. \frac{\partial k_i'}{\partial b_i} \right|_{b=b_j} \approx \frac{k'(z_i, k_i, b_{j+1}) - k'(z_i, k_i, b_{j-1})}{b_{j+1} - b_{j-1}}$$

For the boundary points of the grid, I use forward and backward differences, respectively:

• First point 
$$(b_1)$$
:  $\frac{k'(b_2) - k'(b_1)}{b_2 - b_1}$ 

• Last point 
$$(b_N)$$
:  $\frac{k'(b_N) - k'(b_{N-1})}{b_N - b_{N-1}}$ 

This numerical approach allows for the computation of MPI at every state (z, k, b) without requiring analytical derivatives of the policy function.

### E.2 Weighted Statistics Using the Stationary Distribution

All aggregate statistics (e.g., means, variances, covariances) are computed by weighting each state by its mass in the stationary distribution,  $\mu(z, k, b)$ . This ensures that the statistics reflect the long-run equilibrium distribution of firms.

For any variable X(z, k, b), the weighted mean is:

$$\mathbb{E}[X] = \sum_{z,k,b} X(z,k,b) \cdot \mu(z,k,b), \quad \text{where } \sum_{z,k,b} \mu(z,k,b) = 1$$

The unconditional covariance between MPI and debt is computed as:

$$Cov(MPI, b) = \sum_{z,k,b} \mu(z, k, b) \cdot [MPI(z, k, b) - M\bar{P}I] \cdot [b - \bar{b}]$$

where  $MPI = \mathbb{E}[MPI]$  and  $\bar{b} = \mathbb{E}[b]$  are the weighted means.

This unconditional covariance captures the overall relationship between MPI and debt in the economy. However, it conflates two distinct effects:

- 1. Within-Firm-Type Variation: How MPI changes with debt for a given firm type (fixed z, k). This isolates the Fisherian debt-inflation channel.
- 2. **Between-Firm-Type Variation:** How average MPI and average debt differ across firm types. This captures composition effects.

To disentangle these channels, I decompose the covariance. The **law of total covariance** provides the formal framework:

$$Cov(MPI, b) = \underbrace{\mathbb{E}_{z,k}[Cov(MPI, b \mid z, k)]}_{\text{Within-group component}} + \underbrace{Cov_{z,k}(\mathbb{E}[MPI \mid z, k], \mathbb{E}[b \mid z, k])}_{\text{Between-group component}}$$
(37)

The **within-group component** is the weighted average of conditional covariances. For a given firm type  $(z_0, k_0)$ , the conditional covariance  $Cov(MPI, b \mid z_0, k_0)$  measures how MPI varies with debt, holding productivity and capital fixed. This term isolates the Fisher channel.

The **between-group component** measures the covariance between the conditional mean of MPI and the conditional mean of debt across (z, k) types. This term captures

the composition effect: for example, if highly productive firms tend to have both high debt and low MPI, this component will be negative.

Analogous to the covariance decomposition, the variance of any variable X can be decomposed as:

$$\operatorname{Var}(X) = \underbrace{\mathbb{E}_{z,k}[\operatorname{Var}(X \mid z, k)]}_{\text{Within-group variance}} + \underbrace{\operatorname{Var}_{z,k}(\mathbb{E}[X \mid z, k])}_{\text{Between-group variance}}$$

Applying this to MPI reveals the primary sources of its variation:

- Total variance: 0.260
- Between-group variance (due to z, k): 0.083 (32%)
- Within-group variance (due to b): 0.177 (68%)

Interpretation: The majority (68%) of the variation in MPI arises from differences in debt levels within firm types. This underscores the quantitative importance of the Fisher channel, which operates through this margin. The remaining 32% reflects systematic MPI differences across firms with different productivity and capital levels.

### E.3 Constraint Detection Methodology

To identify which firms are financially constrained, I use a kink-detection method on the capital policy function k'(z, k, b). For an unconstrained firm, the Euler equation is smooth, implying a smooth policy function. When a borrowing or dividend constraint binds, the policy function exhibits a kink.

A binding constraint manifests as a positive second derivative of the policy function with respect to debt,  $\frac{\partial^2 k'}{\partial b^2} > 0$ , indicating that investment becomes more sensitive to debt as the constraint tightens. I compute this derivative numerically using a central difference:

$$\left. \frac{\partial^2 k'}{\partial b^2} \right|_{b=b_j} \approx \frac{k'(b_{j+1}) - 2k'(b_j) + k'(b_{j-1})}{(\Delta b)^2}$$

A firm at state  $(z, k, b_j)$  is classified as constrained if this second derivative exceeds a small threshold of  $10^{-4}$  to account for numerical noise. The share of constrained firms declines sharply with productivity, confirming that high-productivity firms face looser financial constraints:

- Low Productivity  $(z_1)$ : 20.9% constrained
- Medium Productivity  $(z_4)$ : 17.1% constrained

• High Productivity  $(z_7)$ : 2.3% constrained

### E.4 Quantitative Decomposition Results

Table 12 presents the decomposition of the covariance between MPI and debt based on the calibrated model's steady-state distribution.

Table 12: Decomposition of Cov(MPI, b)

Component	Value
Unconditional Covariance: $Cov(MPI, b)$	-0.025
Within-firm-type component (Fisher Channel)	+0.106
Between-firm-type component (Composition Effect)	-0.131
Correlation: $Corr(MPI, b)$	-0.013

The unconditional covariance is small and negative (-0.025), implying that firm heterogeneity slightly dampens, rather than amplifies, the aggregate investment response to an inflation shock. The decomposition reveals a tale of two opposing forces:

- 1. The within-firm-type component is positive (+0.106), confirming that the Fisher channel operates as expected at the micro level: for a given firm type, higher debt is associated with a higher MPI.
- 2. The between-firm-type component is negative and larger in magnitude (-0.131). This reflects a powerful composition effect that offsets the Fisher channel at the aggregate level.

# E.5 Understanding the Negative Composition Effect

To understand the negative between-firm-type covariance, I sort firms by productivity groups.

Figure 10 shows that average debt rises monotonically with productivity: high z firms can borrow more due to higher collateral values. But there is a paradox that despite higher debt, high z firms exhibit lower or similar average MPI compared to medium-z firms. This occurs because high-productivity firms, though highly leveraged, operate far from their constrained regions due to large capital stocks and strong cash flows, making their investment relatively insensitive to net worth changes.

Table 13 quantifies this pattern across productivity groups. High-z firms (12.8% of the population) have average debt of 5.70 but only 4.9% are constrained, with average MPI of

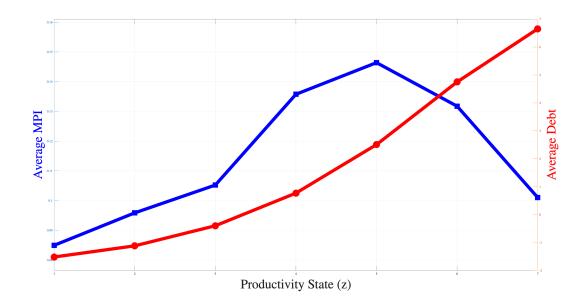


Figure 10: Average Debt and MPI by Productivity Groups

**Notes:** This figure plots the average MPI (blue squares) and average debt (red circles) by productivity state. Higher productivity firms have higher debt but not necessarily higher MPI.

0.117. Low-z firms (12.8%) have average debt of -1.31 (net savers) and 21.9% are constrained, but their average MPI is only 0.091 because many are unconstrained. Medium-z firms (74.4%) have moderate debt (0.68), intermediate constraint rates (16.5%), and the highest average MPI (0.127). The negative correlation between productivity and MPI within debt groups drives the negative between-component (-0.131).

# E.6 Implications for Aggregate Investment

The decomposition results yield three principal implications for the aggregate investment response to an inflation shock.

First, the response is overwhelmingly dominated by the level effect. This channel operates through the mean redistribution of wealth from households to the corporate sector. A simple calculation,  $\mathbb{E}[MPI] \times \mathbb{E}[\text{debt relief}] \approx 0.12 \times 1.22 = 0.146$ , suggests this level effect alone generates a 0.15 percentage point increase in the aggregate investment rate for each 1% inflation shock. This translates to an approximate 0.88% increase in aggregate investment, accounting for the steady-state capital-investment ratio.<sup>15</sup>

<sup>&</sup>lt;sup>15</sup>The exact mapping depends on the aggregate capital stock, which is approximately 6 times the steady-state investment rate in the calibrated model.

Table 13: Firm Characteristics by Productivity Groups

Type	% Firms	Avg Debt	Avg MPI	% Constrained	Corr(z)
Low $z$	12.8%	-1.31	0.091	21.9%	+0.230
Medium $z$	74.4%	+0.68	0.127	16.5%	+0.046
${\rm High}\ z$	12.8%	+5.70	0.117	4.9%	-0.269

**Notes:** High-z firms have high debt (+5.70) but are less constrained (only 4.9%) and have lower MPI (0.117) compared to medium-z firms. Low-z firms have low or negative debt (-1.31), are more constrained (21.9%), but also have low MPI (0.091) because many are unconstrained savers. This generates the negative between-group covariance.

Second, the net heterogeneity effect provides a small, negative drag on the aggregate response. The unconditional covariance of -0.025 arises from two strong but opposing forces. The positive within-firm component (+0.106) confirms the Fisher channel's microlevel validity. However, this is more than offset by a larger, negative between-firm component (-0.131). This composition effect occurs because debt relief flows disproportionately to high-productivity, highly-leveraged firms that, being less financially constrained, exhibit lower MPIs. This mutes the stimulus that would otherwise occur if resources flowed to more constrained firms.

Third, the decomposition highlights a crucial distinction between micro-level mechanisms and macro-level outcomes. The positive within-firm-type covariance (+0.106) is precisely the relationship an empirical strategy using firm-level panel data with firm fixed effects would identify—capturing how a single firm's investment sensitivity changes with its debt over time. However, the aggregate response is shaped by the negative composition effect, which stems from cross-sectional differences in firm characteristics. This underscores the risk of extrapolating micro-level estimates to predict aggregate outcomes without accounting for general equilibrium composition effects.

# E.7 Reconciling with Investment Dynamics

To further validate these steady-state findings, I analyze the characteristics of firms based on their investment response along the model's transitional path following an inflation shock. Table 14 categorizes firms into those that increase ( $\Delta i > 0$ ) and those that decrease ( $\Delta i < 0$ ) their investment.

The table reveals a crucial dynamic: the firms that actually expand investment in response to the shock are not the economy's most productive players. Although they are more productive and more indebted than firms that contract, their average productivity ( $z \approx 0.93$ )

Table 14: Firm Characteristics by Investment Response

	Increasing Investment	Decreasing Investment
	$(\Delta i > 0)$	$(\Delta i < 0)$
Mean $z$	0.9293	0.7629
Mean $k$	4.5754	4.9096
Mean $b$	2.9414	2.5536
Avg MPK	0.0382	0.0226

**Notes:** Firms that increase investment have higher average productivity (z = 0.9293) and debt (b = 2.9414) than those that decrease it. Crucially, both groups have below-average productivity, indicating the Fisher channel's benefits are concentrated among low-to-medium productivity firms.

is below the population mean. This reinforces the decomposition's main insight: while high debt is correlated with high productivity across all firms (the source of the negative composition effect), the marginal impact of debt relief is strongest for the vast population of low-to-medium productivity firms that are both indebted and financially constrained. High-productivity firms may receive the largest wealth transfer, but their low MPI means this transfer does not translate into a significant investment response.

# E.8 Efficiency Implications

A final and critical observation concerns the allocative efficiency of the debt-inflation channel. While unexpected inflation successfully boosts the *quantity* of aggregate investment, it may simultaneously degrade the *quality* of that investment by inducing capital misallocation and reducing aggregate TFP. This occurs through several reinforcing mechanisms.

First, the Fisher channel acts as an untargeted subsidy. It redistributes resources toward indebted, constrained firms, which our model shows are predominantly those with low-to-medium productivity. These are the firms with high MPIs, but they are not the most efficient producers in the economy.

Second, the most productive, unconstrained firms, which are critical for aggregate TFP growth, receive large wealth transfers but exhibit low MPIs and therefore do not respond significantly. This means resources are channeled away from their most efficient uses, worsening the cross-sectional allocation of capital.

Third, even though the overall effect remains negative, the reallocation among responding firms shows a minor efficiency gain. Specifically, firms increasing investment have a higher average productivity than those decreasing it (Table 14). Therefore by comparing these two groups of firms, by transferring resources from decreasing investment firms to increasing investment firms, the misallocation is improved. However, this small, positive reallocation is vastly outweighed by the primary distortion: capital flowing from the general economy to firms that are, on the whole, less productive than the economy's top performers.

Finally, some low-productivity firms that are net savers (b < 0) suffer a wealth loss from inflation. Since they are often unconstrained savers, this loss does not free up any productive investment, resulting in a pure negative wealth effect without offsetting benefits.

Collectively, these forces imply a fundamental trade-off. The debt-inflation channel alleviates firm-level financial frictions, boosting aggregate investment. However, it does so by reallocating capital toward less productive firms, thereby potentially lowering aggregate TFP. This tension between the quantity and quality of investment is a key consideration for evaluating the welfare implications of inflation surprises, a topic I leave for future research.

## F U.S. Sectoral Financial Balances

In this appendix, I document the facts of the US economy and make it clear why in my quantitative model, firms are on aggregate net borrowers while households are net savers. In national accounts, a sector's financial role is determined by the relationship between its savings (S) and its non-financial investment (I). A Net Borrower is a sector where investment exceeds savings (I > S), creating a financial deficit that must be funded by borrowing from other sectors.

This summary analyzes the financial positions of the U.S. household and nonfinancial corporate sectors using the most recent data from Q2 2025, sourced from the Federal Reserve's Financial Accounts of the United States. As of 2025Q2, a significant macroeconomic condition persists: both the household sector and the nonfinancial corporate sector are operating as net savers. The U.S. household sector continues to be the primary source of private savings in the economy. Its balance sheet shows financial assets far exceeding liabilities, confirming its role as a net lender to other sectors.

Table 15: Sectoral Net Lending Summary (2025Q2)

Sector	Net Lending	as % of GDP
Household Sector	+\$652.2 Billion	+2.1%
Nonfinancial Corporate Sector	+\$164.9 Billion	+0.5%

**Note:** Net lending is the seasonally adjusted annual rate for Q2 2025. GDP for Q2 2025 was \$30.49 trillion (SAAR). Sources: Federal Reserve Financial Accounts of the United States (Z.1) and Federal Reserve Bank of St. Louis FRED.

Historically, the nonfinancial corporate sector has been a net borrower, using household savings to fund capital investment. However, a structural shift has occurred, and this sector now operates as a net saver. This phenomenon, often termed the "corporate saving glut," is driven by factors including higher profits due to a secular decline in the labor share Chen et al. (2017), relatively weaker physical investment, the rising importance of intangible assets Falato et al. (2022); Li (2025), and precautionary saving motives Gruber and Kamin (2015); Riddick and Whited (2009).

The fact that both major private domestic sectors are net savers creates a large private sector financial surplus. According to the macroeconomic identity where all sectoral balances must sum to zero, this private surplus must be offset by deficits elsewhere. In the U.S. context, this surplus provides the domestic financing for the combined deficits of the Government Sector, which runs a persistent budget deficit, and the External Sector, as the

U.S. runs a current account deficit, making it a net borrower from the rest of the world.

In Section 4, in a closed-economy version of the model with no government and no external sector, the sectoral financial balances must sum to zero. Writing each sector's net lending as savings minus non-financial investment, we have the identity

Net lending
$$_F = -$$
 Net lending $_H$ .

Thus, within this modeling environment, an aggregate positive net lending position for households mechanically implies an aggregate negative net lending position for firms (i.e., firms are net borrowers), and vice versa. This accounting fact is the reason it is both natural and economically transparent to interpret the model's private-sector flows as a comparison between a (net) saving household sector and a (net) borrowing corporate sector.

This modeling choice aligns with the historical pattern in the U.S., where nonfinancial firms have tended to be net borrowers, using household savings to finance physical capital accumulation and working capital needs. This is the canonical setting for the debt-inflation channel: when firms are net debtors, unexpected inflation erodes the real value of nominal liabilities, relaxing firms' balance sheets and easing investment constraints. In recent decades, however, a structural shift has been documented in many advanced economies, with firms at times becoming net savers due to the factors mentioned above.

Nevertheless, this role reversal matters for the quantitative importance of the debtinflation channel. If firms on aggregate are net savers, the direct redistributive effect of unexpected inflation from creditors to debtors operates differently at the macro level, and the aggregate investment response will generally be attenuated. Because my baseline model omits government and external sectors, I calibrate the model so that in the baseline economy, firms are, on net, aggregate borrowers and households are net savers. This choice isolates the traditional firm-side mechanism emphasized in the paper.